The Role of Numerical Tables in Galileo and Mersenne

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Numerical tables are important objects of study in a range of fields, yet they have been largely ignored by historians of science. This paper contrasts and compares ways in which numerical tables were used by Galileo and Mersenne, especially in the Dialogo and Harmonie Universelle. *I argue that Galileo and Mersenne used tables in radically different ways, though rarely to present experimental data. Galileo relied on tables in his work on error theory in day three of the* Dialogo *and also used them in a very different setting in the last day of the* Discorsi. *In Mersenne's case they represent an important but so far unrecognized feature of his notion of universal harmony. I conclude by presenting a classification of different ways in which tables were used within the well-defined disciplinary and temporal boundaries of my research. In doing so, however, I provide a useful tool for extending similar investigations to broader domains.*

1. The varied role of tables

The topic of this workshop, "Galileo in Paris," provides an ideal opportunity for exploring a number of themes linked to the occurrence and role of numerical tables. As pointed out in a classic essay by Thomas Kuhn, numerical tables are an important if understudied feature of many works in the history of science. Of course, it would be inappropriate here to attempt even an outline of the history of a visual tool spanning over half a millennium. Tables in various forms probably predate the Gutenberg Bible, since almanacs, calendars, and prognostications were printed in all

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likelihood in ephemeral publications before the 1450s. Tables come in many different forms and serve a variety of purposes in many disciplines, from accounting to optics and from the world of insurance to astronomy. My purposes here are largely to do with the mathematical disciplines and especially hydrostatics, astronomy, the science of motion and immediately related fields. I have identified several types of tables, ranging from those used as a computer giving in convenient form the results of tedious computations, to those providing the result of extensive experimental programs.[1]

The following section deals with hydrostatics and shows the care and skill Galileo devoted in specific instances to handling experimental and numerical data. In connection with Galileo and, as it turns out, Mersenne, I examine the work by the mathematician Marino Ghetaldi, who published celebrated tables of the weights of unit samples of several substances. Section (3) deals with Galileo's usage of numerical tables in his refutation of Scipione Chiaramonti's analysis of observational data of the 1572 nova. Galileo's usage of numerical tables in astronomy was linked to a rather sophisticated attempt to use error theory in order to draw plausible conclusions from a conflicting set of data. Section (4) focuses on Galileo's presentation of his science of motion in the *Dialogo* and *Discorsi,* and Mersenne's reactions to it. Especially in *Harmonie Universelle,* Mersenne relied extensively on numerical tables both to examine Galileo's claims and to present his own views. Lastly, I present some tentative conclusions on the different roles and purposes of numerical tables and a conceptual classification of the tables encountered in this work. I hope that scholars will find it a useful starting point for further research.

2. In the wake of Archimedes' crown problem

The first theme I investigate is the determination of the weights of equal volumes of different bodies. We tend almost automatically to talk of specific weight or, to follow medieval terminology, *gravitas in specie.* Several mathematicians such as Galileo and Ghetaldi rigorously adopting the somewhat restrictive theory of proportions, however, had reservations in talking about a ratio between two inhomogeneous magnitudes, such as weight and volume, and preferred to avoid the medieval notion altogether.[2]

Galileo devoted one of his earliest works to the crown problem, his

1. T. Kuhn (1977c); see also Kuhn (1977b) and Dear (1991). Relevant discussions are also in Dear (1988) and (1995). A related area is the history of graphs, for which see T. L. Hankins (1990) and Campbell-Kelly (2003).

2. Napolitani (1988).

1586 *La Bilancetta,* where he tried to find a more plausible method for solving Archimedes' crown problem than that provided by Vitruvius. Galileo claimed that measuring the water that overflowed from the container, as claimed by Vitruvius, was a rather crude method, or "alla grossa." Rather, he argued that a much more precise measurement of the ratio of silver to gold could be attained by means of a hydrostatic balance and Archimedes' propositions on buoyancy. The hydrostatic balance relies on the difference in a body's weight in air and in water. From this difference, compared to those for bodies made of pure gold and pure silver, one obtains the desired ratio of silver to gold in the crown. In order to increase the precision of his measurements, Galileo coiled a thin wire around one arm of the balance. Counting the number of coils allowed a very accurate measure of the distance where the counterweight ought to be hung. Since the coils were too small to be easily counted, Galileo suggested using the sound produced by a pointed probe slowly sliding against them. Therefore sound was used to measure a distance.[3]

Thus one of Galileo's earliest works speaks for his considerable concerns for precision measurement. The context is neither that of establishing a new theory, nor of verifying its conclusions, but rather to provide a more convincing philological account of how Archimedes may have solved the crown problem. Thus in *La Bilancetta* Galileo did not need to provide any numerical data. Although the hydrostatic balance was not his invention, the coiled wire was, and in all probability he used it again. Among his unpublished papers Antonio Favaro found tables of experimental data about precious metals and stones weighed in air and in water. The procedure for measuring the weights of equal volumes of substances appears to be parallel to that of *La Bilancetta,* thus most likely Galileo used the same type of balance he had described there. The nature of the materials studied by Galileo suggests that he had a utilitarian perspective in mind, with goldsmiths and jewelers as an obvious audience for his work. Here we find a concrete example of his interest in numerical data.[4]

Although his tables remained unpublished and, to my knowledge, did not circulate, Galileo found another way of publicizing similar empirical data on this topic by means of the geometric and military compass. While the compass is primarily a calculation device, Galileo included data to determine the size of spheres of equal weight but different materials, such as gold, lead, silver, copper, iron, tin, marble, and stone. Galileo's compass had a practical and utilitarian aim for a range of people including metal

3. Galileo (1890–190), 1:215–20; Drake (1978), pp. 6–7.

4. Galileo (1890–1909), 1:221–8. On experiments in conversation with the classics in a slightly later period see Tribby (1991).

TAVOLA

DELLE PROPORZIONI DELLE GRAVITÀ IN SPECIE

DE I METALLI E DELLE GIOIE

PESATE IN ARIA ED IN AQQUA

Oro puro in aria pesò	grani	$156\ ^{1}/_{4}$	100	1000	576
pesò poi in aqqua	grani	$148\ ^{1}/_{4}$	$94\ ^{22}/_{25}$	$948\ ^{4}/_{5}$	$546\ ^{1}/_{2}$
Argento puro in aria pesò	grani	$179\ ^{1}/_{4}$	100	576	
pesò poi in aqqua	grani	162	$90\ ^{270}/_{717}$	$520\ ^{57}/_{100}$	
altra d'argento più fine		576			
		$520\ ^{95}/_{100}$			
Rame in aria	grani	$179\ ^{9}/_{16}$	576	576	
in aqqua	grani	159	$510\ ^{4}/_{100}$	$510\ ^{36}/_{100}$	
Diamante pesò in aria	grani	$48\ ^{1}/_{6}$	576		
in aqqua	grani	$34\ ^{19}/_{32}$	$413\ ^{63}/_{100}$		
Rubini 3 in aria	grani	$16\ ^{9}/_{16}$	576		
in aqqua	grani	$12\ ^{7}/_{16}$	$432\ ^{54}/_{100}$		
Smeraldo in aria	grani	$133\ ^{7}/_{32}$	576		
in aqqua	grani	$84\ ^{5}/_{32}$	$340\ ^{42}/_{100}$		
Topazio in aria	grani	$381\ ^{1}/_{4}$	576		
in aqqua	grani	$242\ ^{1}/_{2}$	$366\ ^{3}/_{100}$		

4. *pesati* corretto in *pesate* —

Figure 1: Galileo's Tables of metals and precious stones

This table, based on Favaro's edition of Galileo's manuscript, shows the weights of different metals and precious stones measured in air and water. From their difference it was possible to determine the weight of a unit volume of a given substance.

workers and gunners, rather than a foundational one for the new science, yet it documents his interest in quantitative empirical data, one Mersenne would have much appreciated.

It seems appropriate in dealing with a topic like Galileo in Paris to attempt a triangulation with another mathematician who worked on a similar topic, namely Marino Ghetaldi. There are strong reasons for mentioning him here. Ghetaldi, a gentleman from Ragusa, now Dubrovnik, was a friend and correspondent of Galileo, whom he met at Padua in the circle of Gian Vincenzo Pinelli. He too had recourse to the hydrostatic balance, though he does not appear to have used the coiled wire devised by Galileo. In his *Promotus Archimed[e]s* Ghetaldi produced celebrated tables of the weights of several substances that Mersenne greatly admired. He reproduced them in the 1623 *Quaestiones in Genesim,* a monumental work where numerical tables are largely absent, and in the 1644 *Cogitata physico-mathematica.*[5] Ghetaldi included several pages of tables. Those I show here give the ratio between equal volumes of twelve substances, gold, quicksilver, lead, silver, copper, iron, tin, honey, water, wine, wax, and oil.[6]

Ghetaldi did not expand on the time, location, and circumstances of his experimental work, or on the nature of the samples examined, but he provided extensive information on the accuracy of his procedures. He explained that he had trouble obtaining perfect spheres, thus he worked with cylinders with the height equal to the diameter, and then calculated with Archimedes the weight of the inscribed sphere. He printed the unit length of the old roman foot, but in the *Errata* he specified that the paper had shrank, so the unit had to be increased by $^{1}/_{40}$. He highlighted his usage of horse's hairs to hang the weights, since they weigh approximately as an equal volume of water. Ghetaldi relied on special techniques for weighing bodies, such as coating a piece of gold with a thin layer of wax to weigh it in mercury, so it does not amalgamate. He also relied on wax balls attached to lead weights, an idea with some analogies to later used by Galileo in a different context in his dispute with the Aristotelians.[7] In conclusion, the care Ghetaldi claimed he had devoted to weighing procedures was meant to warrant for the reliability of his tables. But of course the tables could be read and used in different ways, as witnesses to the reliability of the data, as convenient unproblematic reference tools to rely on, and also as open to scrutiny and as an invitation to others to check and improve on the data.

5. Mersenne (1623), cols 1155–6 and 1159–60; (1644), pp. 188–92.
6. Ghetaldi (1603), pp. 32ff.
7. Ghetaldi (1603), pp. 9, 24, 34 and 72. On Ghetaldi's work see Napolitani (1988).

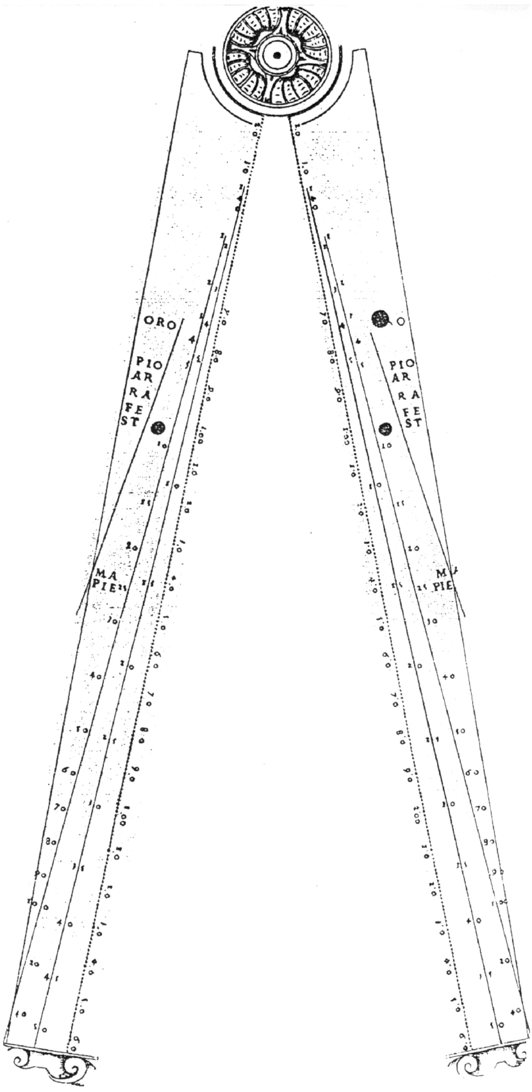


Figure 2: Galileo's geometric and military compass

Galileo's geometric and military compass includes a set of points for determining the sizes of spheres of different materials. Given a sphere of a given material, the compass enables one to determine the diameter of a sphere of another material among those tabulated.

PROMOTVS
Ad comparandum inter ſe duodecim corporum genera grauitate, & magnitudine tabella.

	A urũ.	Ar.Vi.	Plum.	Arg.	Aes.	Ferrũ.	Stann.	Mel.	Aqua.	Vinũ.	Cera.	Oleũ.
Oleum.	$20\frac{3}{11}$	$14\frac{62}{77}$	$12\frac{6}{11}$	$11\frac{3}{11}$	$9\frac{9}{11}$	$8\frac{8}{11}$	$8\frac{4}{55}$	$1\frac{32}{55}$	$1\frac{1}{11}$	$1\frac{4}{55}$	$1\frac{5}{121}$	1
Cera.	$19\frac{19}{21}$	$14\frac{32}{147}$	$12\frac{1}{21}$	$10\frac{52}{63}$	$9\frac{9}{21}$	$8\frac{8}{21}$	$7\frac{89}{105}$	$1\frac{109}{210}$	$1\frac{1}{21}$	$1\frac{13}{420}$	1	
Vinum.	$19\frac{19}{59}$	$13\frac{331}{413}$	$11\frac{41}{59}$	$10\frac{30}{59}$	$9\frac{9}{59}$	$8\frac{8}{59}$	$7\frac{31}{59}$	$1\frac{28}{59}$	$1\frac{1}{59}$	1		
Aqua.	19	$13\frac{4}{7}$	$11\frac{1}{2}$	$10\frac{1}{3}$	9	8	$7\frac{2}{5}$	$1\frac{9}{10}$	1			
Mel.	$13\frac{3}{29}$	$9\frac{73}{203}$	$7\frac{27}{29}$	$7\frac{11}{87}$	$6\frac{6}{29}$	$5\frac{15}{29}$	$5\frac{3}{29}$	1				
Stannum.	$2\frac{21}{37}$	$1\frac{221}{259}$	$1\frac{41}{74}$	$1\frac{44}{111}$	$1\frac{8}{37}$	$1\frac{1}{37}$	1					
Ferrum.	$2\frac{3}{8}$	$1\frac{39}{56}$	$1\frac{7}{16}$	$1\frac{7}{24}$	$1\frac{1}{8}$	1						
Aes.	$2\frac{1}{9}$	$1\frac{12}{63}$	$1\frac{5}{18}$	$1\frac{4}{27}$	1							
Argentum.	$1\frac{26}{31}$	$1\frac{68}{217}$	$1\frac{7}{62}$	1								
Plumbum.	$1\frac{15}{23}$	$1\frac{29}{161}$	1									
Arg. Viuũ.	$1\frac{38}{95}$	1										
Aurum.	1											

Quæro exempli gratia, quam habet rationem in grauitate plumbum ad aurum. Inteligatur plumbum, quoniam leuius eſt auro, grauitatem habere 1, & in linea plumbi in prima columna nominati, ſub titulo auri, quæratur auri grauitas, ea erit $1\frac{1}{2}\frac{5}{7}$. plumbum igitur ad aurum rationem habebit in grauitate vt 1, ad $1\frac{1}{2}\frac{5}{7}$, ſi enim ſumantur duo corpora magnitudine æqualia, vnum plumbeum alterum aureum, ſit autem plumbei corporis grauitas 1, aurei erit $1\frac{1}{2}\frac{5}{7}$, quare corpus plumbeum ad corpus aureum eiuſdem magnitudinis rationem habebit in grauitate vt 1, ad $1\frac{1}{2}\frac{5}{7}$. comparantur autem inter ſe genera diuerſa grauitate, in corporibus magnitudine æqualibus.

Rurſus, quæro quam habet rationem in grauitate aqua ad argentum viuum. inteligatur aqua, vt leuior argento viuo grauitatem habere 1, & in linea aquæ, ſub titulo argenti viui, quæratur argenti viui grauitas, ea erit $13\frac{4}{7}$, aqua igitur ad argentum viuũ rationem habebit in grauitate vt 1, ad $13\frac{4}{7}$.

Contra, quæro quomodo ſe habent in magnitudine aurum, & plumbum. inteligatur aurum, quoniam grauius eſt plumbo, magnitudinem habere 1, & in linea plumbi, ſub titulo auri, quæratur plumbi magnitudo, ea erit $1\frac{1}{2}\frac{5}{7}$, aurum igitur ad plumbum ſe habebit

Figure 3: Tables from Ghetaldi's Promotus Archimedes

Ghetaldi included in his work several numerical tables based on empirical data and direct elaborations on them. The table above enables one to determine the weights of equal volumes of different substances. For example, from the first column, the weights of equal volumes of silver and gold are in the ratio of $1\,^{26}/_{31}$ to 100. Other tables were especially geared to goldsmiths and all those interested in determining the ratio of silver to gold of an alloy.

3. Handling conflicting astronomical data

It is common among Galileo scholars to contrast and compare his methodology and handling of empirical data in his two principal areas of research, the science of motion and astronomy. Winifred Wisan followed this approach in her important study of Galileo's scientific method in a paper originally delivered here in Blacksburg and subsequently published two and a half decades ago. Traditionally issues like Copernicanism, the ephemeredes of the Medicean planets, and the nature and position of sunspots have attracted the lion's share of attention.[8]

Here I focus on Galileo's handling and presentation in tables of numerical observational data on the location of the 1572 nova that appeared in Cassiopeia. This is the only portion of the *Dialogo* where Galileo had recourse to numerical tables. Galileo tackled this problem at the beginning of the third day attacking Scipione Chiaramonti's treatment of observational data in *De tribus novis stellis* (1628), where he had argued that the new star was sublunar. While trying to retrieve useful and reliable information from an array of partly contrasting data, Galileo displayed remarkable acumen and rhetorical skills in dealing with observational error. Although he displayed similar skills and interests elsewhere in his writings, his reflections on the 1572 nova are possibly among the most sophisticated in the seventeenth century.[9] The brief account of the relevant portion of the *Dialogo* allows us to gauge Galileo's sensitivity to observational error.

The problem consisted in determining the position of the star on the basis of the contrasting observations of about a dozen astronomers, notably whether it was located above or below the moon. Chiaramonti selected twelve couples of combinations between observations giving the new star a sufficiently high parallax as to place it below the moon. Galileo's rebuttal is quite complex and extends over dozens of pages of calculations and reflections. Of course, Chiaramonti had selected among possible combinations those that suited him best, but other combinations leading to the opposite conclusion were also possible. Some combinations led to impossible results, such as placing the star very close to or inside the earth, or beyond the fixed stars. Galileo admitted that observational errors were unavoidable for all observations, but he argued that they were more likely to be small rather than large. For example, if "one can modify an obvious er-

8. Wisan (1978).

9. A slightly earlier debate involved monetary estimations, namely if a horse is worth 100 but its value is estimated by two people as 10 and 1,000, who commits the largest error? Galileo's answer was that they were equally wrong. Galileo (1890–1909), 6:565–612, dated 1627. See also Gingerich (1973).

ror and a patent impossibility in one of [the astronomers'] observations by adding or subtracting two or three minutes, rendering it possible by such a correction, then one ought not adjust it by adding or subtracting fifteen, twenty, or fifty minutes."[10] Galileo also argued that impossible values should not be discarded, but had to be corrected, adding that "astronomers, in observing with their instruments and seeking, for example, the degree of elevation of the star above the horizon, may deviate from the truth by excess as well as by defect, that is, erroneously deduce sometimes that it is higher than is correct, and sometimes lower."[11] Further, Galileo pointed out that the size of the mistakes in determining the star's parallax is not proportional to the corresponding mistakes in the star's position, because tiny errors of parallax could result in huge changes in the star's position, as when parallax is small. Moreover, he attacked Chiaramonti's presentation of data for reasons associated with orders of magnitude. In one instance Chiaramonti had determined a star's distance as $373{,}807\,{}^{211}/_{4097}$ miles. "Now, argued Salviati/Galileo, when I am quite sure that what I seek must necessarily differ from correctness by hundreds of miles, why should I vex myself with calculations lest I miss one inch?"[12] These are quite impressive observations not frequently found in print at the time.

The climax of Galileo's attack is a lengthy series of calculations showing the corrections required to make the twelve couples of observations selected by Chiaramonti point to the star's distance from the earth being thirty-two radii. That value was selected as the closest to the moon's orb among those found, or the most favorable to Chiaramonti. Galileo's calculations are summarized in a table of corrections, showing that from the sum of all the parallaxes, giving 836', we have to make corrections in excess of 756'. By contrast, Galileo could show that he could select an equal number of couples of observations requiring corrections of $10'\,{}^{1}/_{4}$ to make the star in the firmament.[13] His table is based on computations but it relies ultimately on observational data, which are subsequently manipulated for ostensive purposes.

Galileo's analysis reveals the care he had devoted to error analysis in astronomy and the sophistication of his reflections, by seventeenth century standards. Sagredo concludes Salviati/Galileo's lengthy demolition of Chiaramonti's work with one of the most devastating passages of the

10. Galileo (1890–1909), 7:331–4, quotation at 314. See also Drake (1967), pp. 289–90.

11. Galileo (1890–1909), 7:316; Drake (1967), p. 291.

12. Galileo (1890–1909), 7:321; Drake (1967), p. 296. Galileo makes an analogous claim in the *Discorsi,* 8:109.

13. Galileo (1890–1909), 7:34.

1ª, del Maurolico e dell' Hainzelio; onde si raccoglie, la stella essere stata lontana dal centro manco di 3 semidiametri terrestri, essendo la differenza di parallasse gr. 4. 42 m. p. e 30 sec. .	3	semidiametri;
2ª, e calculata dall'osservazioni dell' Hainzelio e dello Schulero, con parallasse 8 m. p. e 30 sec.; e si raccoglie la sua lontananza dal centro più di	25	semidiametri;
3ª, e sopra le osservazioni di Ticone e dell' Hainzelio, con parallasse di 10 m. p.; e si raccoglie la distanza dal centro poco meno di. . .	19	semidiametri;
4ª, e sopra l'osservazioni di Ticone e del Landgravio, con parallasse di 14 m. p.; e rende la distanza dal centro circa	10	semidiametri
5ª, e sopra l'osservazioni dell' Hainzelio e di Gemma, con parallasse di 42 m. p. e 30 sec.; per la quale si raccoglie la distanza circa.	4	semidiametri
6ª, e sopra l'osservazioni del Landgravio e del Camerario, con parallasse di 8 m. p.; e si ritrae la distanza circa	4	semidiametri
7ª, e sopra l'osservazioni di Ticone e dell' Hagecio, con parallasse di 6 m. p.; e si raccoglie la distanza	32 [1]	semidiametri;
8ª, e con l' osservazioni dell' Hagecio e dell'Ursino, con parallasse di 43 m. p.; e rende la distanza della stella dalla superficie della Terra	1/2	semidiametro
9ª, e sopra le osservazioni del Landgravio e del Buschio, con parallasse di 15 m. p.; e rende la distanza dalla superficie della Terra	1/48	di semidiametro
10ª, e sopra l'osservazioni del Maurolico e del Munosio, con parallasse di 4 gr. e 30 m. p. [2]; e rende la distanza dalla superficie della Terra	1/5	di semidiametro
11ª, e con le osservazioni del Munosio e di Gemma, con parallasse di 55 m. p.; e rendono la distanza dal centro circa.	13	semidiametri
12ª, e con le osservazioni del Munosio e dell' Ursino, con parallasse di gr. 1 e 36 m. p.; e si ritrae la distanza dal centro meno di.	7	semidiametri

Figure 4: Observations of the 1572 nova

Galileo tabulated twelve couple of observations of the 1572 nova, giving the parallax and the distance from the earth in semi diameters of the earth. According to all the calculations the new star would be sublunary, since its greatest distance would be thirty-two earth radii.

Queſte ſono le correzioni delle Parallaſſe delle 10. *indagini dell'autore, per ridur la ſtella in altezza di* 32. *ſemid.*

Gr. 1.11.	Gr. 1.11.
4.22.*m.p. e* 30. *ſec.*	*ſop.* 4.42.30.
4.	*ſop.* 0.10.
10.	*ſop.* 0.14.
37.	*ſop.* 0.42.30.
7.	*ſop.* 0.18.
42.	*ſop.* 0.43.
14. *e* 50. *ſec.*	*ſop.* 0.15.
4.28.	*ſop.* 4.30.
35.	*ſop.* 0.55.
1.16.	*ſop.* 1.36.

216.	296.	60.
540.	540.	9.
756.	836.	540.

Di qui ſi vede, come per ridur la ✱ *all'altez. di* 32. *ſemid. biſogna dalla ſomma delle Parall.* 836. *detrarne* 756. *e ridurle a* 30. *nè aco baſta tal corez.*

Figure 5: Galileo's table of corrections

This unusual table gives the corrections necessary to make all observations agree that the new star's distance is thirty-two radii. Galileo omitted from the previous table the seventh value, giving exactly thirty-two radii, and also the second, giving twenty-five radii, since the correction of only 1'30" is negligible. His calculations contain small errors.

Dialogo: "This is as if I were watching some unfortunate farmer who, after having all his expected harvest beaten down and destroyed by a tempest, goes about with pallid and downcast face, gathering up such poor gleanings as would not serve to feed a chick for one day."[14]

4. Testing the science of motion

Having briefly surveyed examples from hydrostatics and astronomy, I believe we are now better placed to examine Galileo's presentation of the science of motion and his contemporaries' reactions to it. Galileo's manuscripts and correspondence reveal his skill in analyzing experimental data and sophistication in reducing observational errors, yet his published texts reveal little of his reflections and acumen. Some of his Padua manuscripts and his 1639 letter to Baliani are especially significant.[15]

Ample portions of the science of motion are discussed in the *Dialogo,* but the formal presentation of the new science occurred only in the *Discorsi,* where he introduced his treatise in axiomatic form in Latin, interspersed with elucidations in Italian. In the *Dialogo* Galileo adopted a discursive approach in Italian, discussing experiments in a rather informal fashion.[16] It is here, however, that he gave some numerical values that attracted the attention of his contemporaries. In the second day Galileo states that "per replicate esperienze" a ball of one hundred pounds falls one hundred *braccia* in five seconds, a distance that is roughly half the correct one. In this area Galileo was primarily interested in establishing proportions between variables such as distances, times, and speeds, rather than finding numerical values or parameters. No such value is presented in the *Discorsi.* This is the sort of problem, however, that appealed to Mersenne and to the Genoese patrician Gianbattista Baliani, who in 1632 inquired with Galileo how to determine his value, since he lacked buildings of sufficient height. At the time Galileo had more pressing concerns, but in 1639, following a second request by Baliani, he replied that he had used the inclined plane, thus he had not performed the experiment directly. As to time, with the help of some friends they counted the number of oscillations of a pendulum of arbitrary length necessary for a star to return to its original position in twenty-four hours, and then found the number of seconds by means of the relation between periods of oscillations and dis-

14. Galileo (1890–1909), 7:346; Drake (1967), p. 318. I have changed chicken to chick the Italian being *pulcino.*

15. Recent analyses of Galileo's manuscripts in volume 72 of the Galilean collection at the Florence National Library are in P. Damerow (1992); Renn (2001). Galileo (1890–1909), 18:75–9 and 93–5; Galileo to Baliani, August 1 and September 1, 1639; Drake (1978), pp. 398–404.

16. In the vast literature on the *Dialogo* see Jardine (1991).

tances. Despite giving all these details, Galileo seemed reluctant to commit himself to the value he had provided, arguing that it may need revision. In fact, in a marginal addition to his own copy of the *Dialogo* he did provide a much-improved value. He had Simplicio say that a lead ball of one hundred pounds would fall more than one hundred *braccia* in four pulse beats, a much more accurate value that would have been greatly welcome by Mersenne.[17] It seems plausible that Galileo had the experiment tried after the 1639 letter by Baliani, this time directly from a high tower as opposed to by extrapolation from an inclined plane.

Galileo's correspondence with Baliani is useful in interpreting his published data and Mersenne's problems with them. Galileo's value was less than excellent and it was affected by a major error. By having bodies roll down inclined planes, part of their speed was taken up by the rotation. For a sphere the distance traveled down an inclined plane is $^5/_7$ of what one may have expected by extrapolation from free fall. The problem does not arise, however, if one compares bodies rolling along planes with different inclinations.[18] Much like Baliani, Mersenne too had trouble with Galileo's values. Besides the as yet unclear problem of rolling, his trouble was also due to unit measures. He first assumed that a Florentine *braccio* was twenty inches, then Peiresc told him that is was $21^1/_2$. Several years later in Rome, in a little shop near St. Peter's selling unit measures for the whole of Italy, Mersenne found that the correct value was 23 inches. Such problems with units were of major concern to him. Even so, Galileo's published value was far too low and puzzled him.[19]

In 1634 Mersenne published two works directly related to Galileo, a French translation of *Le mecaniche* and a short pamphlet in the wake of Galileo's claims on falling bodies in the *Dialogo, Traité des mouvemens.* Mersenne's translation contains several additions, yet he included no numerical tables. In the *Traité* Mersenne, in a radical departure from his pre-

17. Galileo (1890–1909), 7:250; Galileo (1890–1909), 14:342; Baliani to Galileo, 23 April 1632; 18:75–9; Galileo to Baliani, 1 August 1639. For the marginal addition in Galileo's own copy see Galileo (1998), vol. 1, p. 32.

18. It has long been a mystery when this problem was first adequately conceptualized. Although Mersenne detected it, I suspect that it was Huygens who became aware that the speeds of bodies of different shapes, a ring, a cylinder, a sphere, and a prism on rollers, rolling down an inclined plane were different. Huygens was also the one who found the solution to the problem of the center of oscillation. The two problems are related and both rely on what we call moment of inertia of a body. See Huygens (1888–1950), 19:158.

19. Mersenne (1636), p. 88; (1647), p. 192. Mersenne opened *Cogitata physico-mathematica,* pp. 1–40, with an extensive discussion of unit measures of distances and weights, including the weights of coins. He compared Roman, German, Hebraic and other units with Parisian ones and on page 2 even established with the help of a microscope how many grains of the sand of Étaples, on the English Channel, make a Parisian foot.

vious views, claimed to have confirmed by means of very accurate experiments the proportionality between distances and the square of the times. Mersenne reported that a lead ball descends 147 feet in $3^1/_2$ seconds, 108 feet in 3 seconds, and 48 feet in 2 seconds. His values were somewhat low, but better than Galileo's published ones. On the basis of these data he constructed several tables, such as the following one.[20]

In his tables he made the significant assumption that the proportion established is invariant under the choice of different unit measures of time.[21] Mersenne's table is based at the very best on three experimental data and relies on huge extrapolations ranging, as Mersenne explains, over all the heights and depths one could encounter. Of course, even the three experimental values arouse suspicion, since they show a perfect agreement down to the last half-second and foot. Here Mersenne was not at all interested in giving raw experimental data, but rather in providing an ordered array of numbers loosely tied to experiment so one could determine heights from the time of fall. Mersenne enjoyed presenting and discussing numerical values in general, and he relished in particular the quasi-aesthetic pleasure of ordered columns and rows.

Mersenne's brief pamphlet was later incorporated in *Harmonie Universelle,* a sprawling treatise dealing with the problem of musical harmony broadly conceived and covering a wide range of subjects, such as the nature and properties of sound and the features of string instruments. The book is a bibliographic nightmare with so many errors that it has been alleged that no two identical copies exist. For this reason it has become customary to rely on the 1965 reprint of Mersenne's own copy with his own annotations.[22] The second book, as Mersenne explained in the introductory *Advertissement,* is devoted to an examination of the *Dialogo* by Galileo, "a most excellent philosopher," in order to establish some principles useful in physics.[23] Indeed, Mersenne scrutinized many propositions by Galileo, both mathematical and experimental, such as the problem of extrusion of a body on a rotating earth and the motion of pendulums. The reason for dealing with Galileo was that sound is motion and Galileo's *Dialogo* deals with motion.

Readers seeking tables in Mersenne's treatise will feel overwhelmed by

20. Dear (1988), chapters 4 and 5 and pp. 136–7. Mersenne's pamphlet, *Traité des mouvemens, et de la cheute des corps pesans, & dela proportion de leurs differentes vitesses* (1634), is extremely rare and has been reprinted in *Corpus: Revue de philosophie* (1986), pp. 25–58, at 36.

21. I doubt whether he realized the consequences of this implication, as several other scholars noticed in the same years. This matter is explained below.

22. Mersenne (1963). For bibliographic information see pp. vii-viii.

23. Mersenne (1963), p. 84.

TABLE DES CHEVTES.

I	II	III	IV
1	1	3	3
2	3	9	12
3	5	15	27
4	7	21	48
5	9	27	75
6	11	33	108
7	13	39	147
8	15	45	192
9	17	51	243
10	19	57	300
11	21	63	363
12	23	69	432
13	25	75	507
14	27	81	588
15	29	87	675
16	31	93	768
17	33	99	867
18	35	105	972
19	37	111	1083
20	39	117	1200
21	41	123	1323
22	43	129	1452
23	45	135	1587
24	47	141	1728
25	49	147	1875
26	51	153	2028
27	53	159	2187
28	55	165	2352
29	57	171	2523
30	59	177	2700

Figure 6: Mersenne table of falls

Mersenne's table does not provide experimental data, but rather is an extrapolation linking the times of fall, in half-seconds, and the distance fallen. The first column gives the half seconds, the second column the sequence of odd numbers, the third column gives how many feet the ball falls in each time interval, whereas the last gives the total distance fallen.

the richness of their trove. Besides the few retrieved from the 1634 treatise, one finds many others dealing with several issues, including the problem of motion. In a particularly interesting one he compared different theoretical proportions of fall, showing against hasty extrapolations that over short distances it is not easy to discriminate among them empirically.[24]

No other place provides more revealing evidence of Mersenne's attitude to tables than his criticism of Galileo's doctrine of falling bodies along inclined planes. In the first day of the *Dialogo* Galileo had argued that given an inclined plane ABC with a side CB perpendicular to the horizon, a body takes the same amount of time to fall along the vertical CB, as it takes to fall on the portion CT of the incline from the vertex C to the perpendicular from B to the hypotenuse. In all likelihood Galileo reached his conclusion from theoretical premises and extrapolations from inclined planes with low inclinations.[25]

Mersenne was not one to take such statements at face value and experimented with inclined planes of 15º, 25º, 30º, 40º, 45º, 50º, 60º, 65º, and 75º. Two equal spheres of lead or wood were released at the same time, while one fell vertically from a height of five feet, the other rolled down the inclined plane. At all elevations Mersenne found that the rolling sphere covered a distance noticeably shorter than that predicted by Galileo. Most values gave a difference of at least half a foot. However, here we do not find a table comparing expected results with experimental data. This was not one of the tasks Mersenne's tables aimed to accomplish, yet I, and I guess most modern readers, would feel strongly tempted to present the results of his experiment in tabular form as a "natural" layout. Indeed, while reading it I found myself constructing a table to figure out what was going on.[26]

In *Harmonie universelle* Mersenne doubted that Galileo had performed experiments with inclined planes since the ratio between rolling and falling was not as expected, but in a marginal note handwritten in his own copy of the book he referred to Galileo's account of the inclined plane experiment in *Discorsi.*[27] Let us move to that work.

The first two days of the *Discorsi* deal primarily with the science of the

24. Mersenne (1963), p. 126. Dear (1995), p. 132, stresses Mersenne's concern with experience, even if it is repeated many times.

25. Galileo (1890–1909), 7:49–50.

26. Mersenne (1963), pp. 111–2. For 15° the sphere rolled down 12" instead of the expected 16"; for 25°, 18" instead of 25"; for 30°, 24" instead of 30"; for 40°, 33" instead of 38"½; for 45°, 36" instead of 42"; for 50°, 33" instead of 46"; for 60°, 33" instead of 52"; for 65°, 36" instead of 54"; for 75°, 42" instead of 58." Dear (1995), pp. 131–2, emphasizes Mersenne's narrative style.

27. Mersenne (1963), p. 112.

Table de cheutes

	I	II	III	IV	V
1	3	3	5	5	3
2	9	12	8	13	5
3	15	27	13	26	8
4	21	48	21	47	13
5	27 33	75	34	81	21
6	39	108	55	136	34
7	45	147	89	225	55
8	~~51~~	192	94	319	89

Figure 7: Table from Mersenne, Harmonie Universelle, 126

Mersenne constructed this table as a warning against careless extrapolations. Several laws of fall may resemble each other over a short distance, only to diverge dramatically later on. Mersenne added some corrections by hand in his own copy of the book.

resistance of materials, but they also contain many digressions, including one on the pitch of musical strings in relation to their length, thickness, and tension. Unlike Mersenne, however, Galileo did not use numerical tables, but geometric diagrams showing the lengths of different strings.[28]

In the *Discorsi* the deductive structure of Galileo's science poses constraints on the role and presentation of experiments. Recent works on the seventeenth century have focused on narrative styles and conventions and on the problems of relying on contrived experiments produced at certain times and places to make universal claims about nature.[29] However, narrative styles were affected by a number of factors, such as the specific mathematical discipline involved and the particular way a discipline was formulated. I have in mind the formulation of a discipline in axiomatic form

28. Galileo (1890–1909), 8:138–49. Mersenne's usage of tables to tune a string instrument is discussed below.

29. A *locus classicus* is Schmitt (1969). See also, Dear (1995); Daston (1991). Of course, the specific features of how the experiment was performed, its reliability and power of persuasion were also highly relevant.

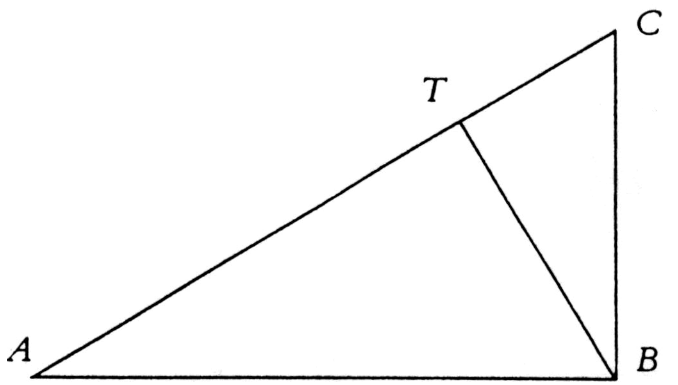


Figure 8: Fall along an inclined plane
Galileo argued that a body falling along the vertical CB would require the same amount of time to fall along CT on the incline, where BT is perpendicular to AC. However, since a body on the incline CTA normally rotates, it will be somewhat slower depending on its shape and will not reach T.

following an Archimedean pattern, as is the science of motion in Galileo's *Discorsi* for example, versus a looser formulation where various results are tested and presented piecemeal, as in the *Dialogo* or Mersenne's *Harmonie Universelle.*

In the *Discorsi* Galileo's science of motion was dependent on a definition and a principle or postulate. The definition states that uniformly accelerated motion from rest is that where speeds are proportional to the times. The principle states that the "gradus velocitatis" acquired by a body in falling along inclined planes with different inclinations are equal whenever the heights of the inclined planes are equal. Galileo had Sagredo argue that his "lume naturale" or innate understanding made him accept the principle, but Salviati presented an additional experiment in its support, namely pendular oscillations. The bob raises back to its original height even if one was to interpose a nail along the vertical, forcing it to move on a different path. The same would be true, according to Galileo, if a sphere was to fall along a straight line as on an inclined plane, rather than an arc, as in the pendulum. In theorem 2 and its corollary, Galileo proved that distances traversed with uniformly accelerated motion are as the squares of the times, whence follows the odd-number rule. Now follows the famous experiment with the inclined plane that attracted Mersenne's attention in his marginal note. The experiment was meant to show that Galileo's mathematical construction was applicable to the real world, since falling bodies follow the odd-number rule. Thus this experiment was not really presented as having a foundational role, since Galileo's mathematical construction, as opposed to its empirical success, would not be affected by a negative result.[30]

It is instructive to contrast the subsequent attitude of Galileo and his closest associates to the postulate and the odd-number rule. Vincenzo Viviani pressed his master over the issue of the postulate until Galileo, after the publication of his treatise, came up with a solution in the form of a proof of his assumption, which was subsequently published in the second edition of the *Discorsi* in 1656. As to the odd-number rule, several mathematicians like Galileo himself, Torricelli, Huygens, and Le Tenneur, as well as the physician Theodore Deschamps, defended its superiority against rival versions due to the Jesuits Le Cazre and Fabri, for example, by arguing that it is invariant under the choice of different time-units, whereas its rivals were not. Thus in both cases there was an attempt not to provide more accurate experimental results and to present them in numerical tables, but rather to reduce altogether the role of experiments by invoking what we may call a regulative or architectonic principle of reason. Here the problem of establishing an axiomatic mathematical theory was not tied to different narrative strategy of how experiments were performed, but to finding suitable principles.[31]

30. Dear (1995), p. 127, somewhat puzzlingly claimed that the odd-number rule was "the basic assumption" of Galileo's science of motion. This claim is not true for the specific axiomatic presentation chosen by Galileo, although here I believe Dear had in mind Galileo's broader concerns rather then his specific presentation. Similar comments apply also to p. 143: "Galileo could not achieve the requisite effect of indubitability for his first principles, however, insofar as he could not be sure of their universal acceptance by his expected readership. He therefore threw in his inclined plane trials ... as a means of establishing one of his fundamental theorems." This statement may apply to the claim that bodies falling along inclined planes can rise to their original height, not to the inclined-plane experiments relevant to the odd-number rule as used by Galileo in *Discorsi.*

31. Galileo (1890–1909), pp. 214–9. Fabri, for example, denounced the odd-number rule and proposed instead the series of natural numbers, 1, 2, 3, 4 etc. By changing the unit of time, doubling it for example, the odd-number rule gives the sequence 4(=1+3), 12(=5+7), 20(=9+11), etc., proportional to 1, 3, 5, etc., therefore if the distance fallen in the first time interval is as 1, those fallen in subsequent intervals are as 3, 5, etc. By contrast, changing the time units changes Fabri's sequence beyond recognition. In this sense Galileo's rule is invariant for a change of units of time, whereas Fabri's is not. Huygens (1888–1950), 1:24–8; Huygens to Mersenne, Leyden, 28 October 1646. Mersenne (1932–88), 12:351; Deschamps to Mersenne, 1 November 1643. Galileo had already outlined a similar reasoning in his letter to Baliani of 20 February 1627, Galileo (1890–1909), 13:348–9. Torricelli (1919), 3:326–8, at 326–7; Torricelli to Mersenne, June 1645, pp. 461–6, at 461–2; Torricelli to Giovanni Battista Renieri, August 1647. See also Palmerino (1999), pp. 269–328, at 295–7 and 319–24 (on Le Tenneur); (2003). *Archimedes,* vol. 6, pp. 187–227. Lenoble (1943), pp. 423–5. Other examples include the

DEL GALILEO. 28

Tabula continens Altitudines, & ſublimitates Semiparabolarum, quarum amplitudines eædem ſint, partium ſcilicet 10000, ad ſingulos gradus Elevationis calculata.

Gr.	Altit.	Subli.	Gr.	Altit.	Subl.
1	87	286533	46	5177	4828
2	175	142450	47	5363	4662
3	262	95802	48	5553	4502
4	349	71531	49	5752	4345
5	437	57142	50	5959	4196
6	525	47573	51	6174	4048
7	614	40716	52	6399	3906
8	702	35587	53	6635	3765
9	792	31565	54	6882	3632
10	881	28367	55	7141	3500
11	972	25720	56	7413	3372
12	1063	23518	57	7699	3247
13	1154	21701	58	8002	3123
14	1246	20056	59	8332	3004
15	1339	18663	60	8600	2887
16	1434	17405	61	9020	2771
17	1529	16355	62	9403	2658
18	1624	15389	63	9813	2547
19	1722	14522	64	10251	2438
20	1820	13736	65	10722	2331
21	1919	13024	66	11230	2226
22	2020	12376	67	11779	2122
23	2123	11778	68	12375	2020
24	2226	11230	69	13025	1919
25	2332	10722	70	13237	1819
26	2439	10253	71	14521	1731
27	2547	9814	72	15388	1624
28	2658	9404	73	16354	1528
29	2772	9020	74	17437	1433
30	2887	8659	75	18660	1339
31	3008	8336	76	20054	1246
32	3124	8001	77	21657	1154
33	3247	7699	78	23523	1062
34	3373	7413	79	25723	972
35	3501	7141	80	28356	881
36	3633	6882	81	31569	792
37	3768	6635	82	35577	702
38	3906	6395	83	40222	613
39	4049	6174	84	47572	525
40	4196	5959	85	57150	437
41	4346	5752	86	71503	349
42	4502	5553	87	95405	262
43	4662	5362	88	143181	174
44	4828	5177	89	286499	87
45	5000	5000	90	infinita.	

Nn PRO

Figure 9: Galileo's tables from the Discorsi

In *Discorsi* Galileo included tables related to parabolic trajectories. The one above gives the amplitude of the semi-parabola described by a body shot with the same speed for different angles of elevation. Notice that the greatest amplitude is for 45°, while complementary angles (whose sum is 90°) give the same amplitude.

denial of perpetual motion (adopted by Stevin), Galileo's notion that a pendulum bob raises to its original height, Torricelli's principle about the center of gravity of combined bodies having to descend for them to move, and Huygens' relativity of motion in the investigation of impact. Huygens was especially keen to adopt different versions of Torricelli's principle. In these cases experiments had largely a private heuristic role and a public ostensive one at best. The situation was quite different for Galileo in earlier stages of the formulation of his science and for other scholars.

At the end of the *Discorsi* and in *De motu* Galileo and Torricelli added numerical tables providing data about parabolic trajectories with different elevations. Those tables, however, contain no experimental data, but resemble standard trigonometric tables appearing in astronomical works. Mersenne reproduced Galileo's tables in his 1644 *Cogitata physico-mathematica.*[32]

Under the narrow compass of this paper I have been able to show and discuss, albeit briefly, all the tables in Galileo's two chief publications. Whilst it is conceivable to provide a comprehensive account of numerical tables in Galileo,[33] this task is beyond my ability for Mersenne. No manageable compass, however large its aperture, could have included even a representative portion of the tables in *Harmonie Universelle.* This seems to be an important and perhaps under-analyzed feature of his work, one revealing of his philosophical perspective on nature and emphasis on the harmony of the world.

5. Concluding reflections

Mersenne's lack of interest in axiomatic formulations and passion for music and harmony gave him a special perspective of the mathematical disciplines.[34] In *Harmonie universelle* he felt the need to present a table of the weights to be attached to a string producing the first audible sound with a weight of six pounds, in order to produce forty-two octaves, arguing that the weight of the earth would only suffice to reach forty-one octaves.[35] Thus Mersenne selected a physically significant value, despite the fact that one could not experiment with it. It appears that he was interested in presenting tables covering a realm in nature in a fashion only partly and loosely linked to experience. Mersenne took an aesthetic and intellectual pleasure both in numbers and in their tabulations. His infatuation with tables was linked especially to *Harmonie universelle,* and to a smaller extent to *La verité des sciences,* while in other works from *Quaestiones in Genesim* to *Cogitata physico-mathematica* tables appear much more sporadically. In *Harmonie universelle,* outdoing Jonathan Swift, Mersenne went as

32. Galileo (1890–1909), 8:304–8. Mersenne (1644), pp. 104 and 108.

33. I am not considering here tables of the Medicean planets since they were presented as diagrams rather then arrays of numbers.

34. The works of Johannes Kepler are an extraordinary source of tables, and one especially interesting to compare to Mersenne. Kepler titled one of his books *Harmonice mundi* (1619). It is well known that the notion of harmony had deep theological implications for both scholars.

35. Mersenne (1636), pp. 184–9, at 186–7. Mersenne talks of the harmony of the seven planets and the earth by considering the sound produced if they were appended to eight equal strings.

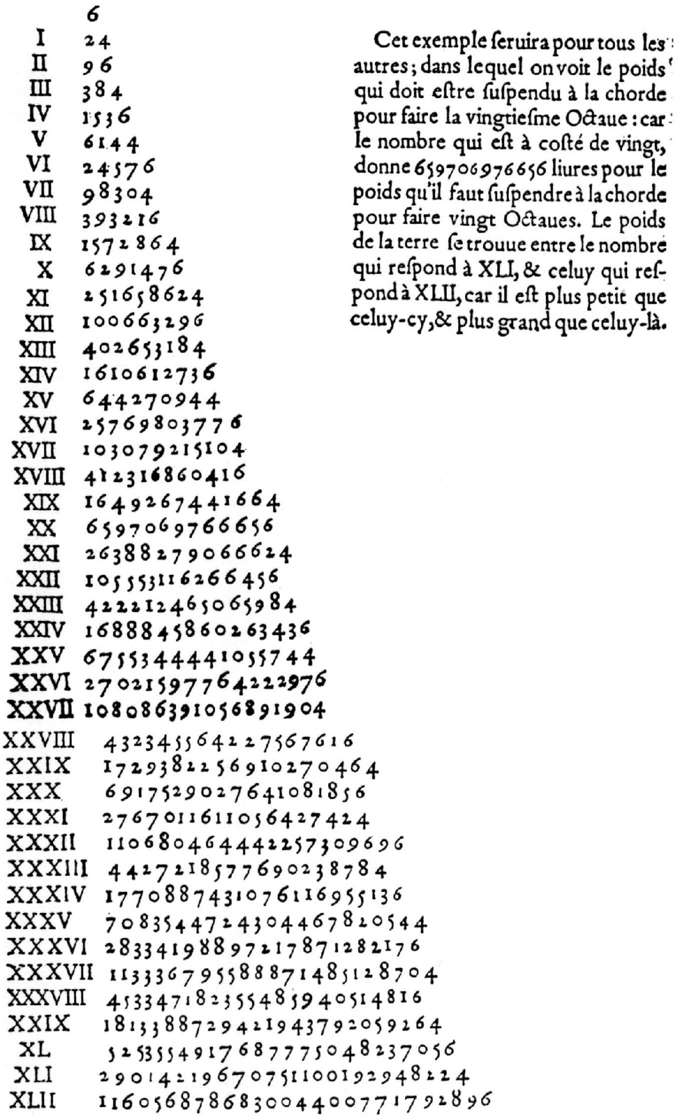
far as to present a table enabling a deaf person to tune a string instrument. Ironically, the table is incorrect because in talking of the *grosseur* of a string Mersenne shifted from its linear diameter to its cross section, but even this quixotic attempt is revealing of the crucial role of ordered number sequences in understanding harmony. The deaf person is capable not simply of tuning the string instrument operationally, but also to appreciate harmony as manifested in the table.[36]

Galileo's tables and usage of numerical data present different features. Galileo's interest in the (specific) weights of different substances seems based on utilitarian aims, whereas in the case of the 1572 nova observational data had major philosophical implications on its location. In the science of motion, however, Galileo saw proportions as the primary mathematical language of nature. His science of motion included theorems and a number of geometrical problems, but in this context determining numerical values was a curiosity devoid of deep philosophical and scientific interest to him.

When we think of a numerical table in the mixed or physico-mathematical disciplines we often have in mind one comparing theoretical computations with experimental data. Such tables were used to argue that the discrepancies were very small, or had an eye to explaining secondary effects emerging from those discrepancies. However, this is not what we tend to find in the published works by Galileo and Mersenne, though we may find it among Galileo's manuscripts as a private heuristic device. To remain within the science of motion, we find extensive tables with numerical data of experimental results on falling bodies with the Jesuit mathematician Gianbattista Riccioli. His enormous *Almagestum novum* (1651) relies extensively on tables on a remarkable range of topics and with remarkably different aims. Even within the comparatively few pages on falling bodies, tables are used in many ways, including the presentation of experimental data on bodies of various materials and sizes falling from the Asinelli tower. His data showed that in no case among the twenty-one investigated did the spheres fall exactly at the same time.[37] Riccioli tried to provide an Aristotelian explanation for the discrepancies in the dis-

36. My favorite table in Mersenne, *La verité des sciences* (1625), is at pp. 549–51, where one finds the number of combinations of tunes that can be produced by an instrument with up to fifty strings. Mersenne (1636), p. 125. See also Dostrovsky (1975), pp. 169–218, at 186–7.

37. Riccioli (1651), vol. 2, pp. 387b-388a. See also Koyré (1968), pp. 89–117; Dear (1995), pp. 71–85; at 82 Dear reproduces one of Riccioli's tables showing that the vibrations of the pendulum are isochronous and that a clay sphere of eight ounces falls in accordance with the odd-number rule; on the experiment discussed in the text see pp. 83–4; on mathematical tables in the second half of the century see pp. 201–9.



	6
I	24
II	96
III	384
IV	1536
V	6144
VI	24576
VII	98304
VIII	393216
IX	1572864
X	6291476
XI	251658624
XII	100663296
XIII	402653184
XIV	1610612736
XV	644270944
XVI	25769803776
XVII	103079215104
XVIII	412316860416
XIX	1649267441664
XX	6597069766656
XXI	26388279066624
XXII	105553116266456
XXIII	422212465065984
XXIV	1688845860263436
XXV	6755344441055744
XXVI	27021597764222976
XXVII	108086391056891904
XXVIII	432345564227567616
XXIX	1729382256910270464
XXX	6917529027641081856
XXXI	27670116110564327424
XXXII	110680464442257309696
XXXIII	4427218577690238784
XXXIV	1770887431076116955136
XXXV	7083544724304467820544
XXXVI	28334198897217871282176
XXXVII	113336795588871485128704
XXXVIII	453347182355485940514816
XXIX	1813388729421943792059264
XL	5253554917687775048237056
XLI	29014219670751100192948224
XLII	116056878683004400771792896

Cet exemple ſeruira pour tous les autres; dans lequel on voit le poids qui doit eſtre ſuſpendu à la chorde pour faire la vingtieſme Octaue: car le nombre qui eſt à coſté de vingt, donne 6597069766656 liures pour le poids qu'il faut ſuſpendre à la chorde pour faire vingt Octaues. Le poids de la terre ſe trouue entre le nombre qui reſpond à XLI, & celuy qui reſpond à XLII, car il eſt plus petit que celuy-cy, & plus grand que celuy-là.

Figure 10: Mersenne's table, Harmonie, I 186–7

Mersenne's remarkable table gives the octaves produced by a string with different weight attached. The first audible sound is produced by a weight of six pounds; the first octave requires twenty-four pounds, and so on to the forty-second octave. The weight of the earth is not sufficient to reach the forty-second octave.

Tablature harmonique pour les sourds.

	Les 8 sons, ou notes de l'Octaue.	Les 7 degrez de l'Octaue.	Table I. La tension des chordes proportionnées selon la raison doublée des interualles.				Table II. La grosseur des chordes proportionnée selon la raison simple des interualles.			Table III. La longueur des chordes proportionnées selon la raison simple des interualles.			Table IV. La Tension des chordes proportionnées selon la raison simple des interualles.			
			liures.	onces.	gros.	grains.	de ligne	parties	dixiémes.	pieds.	poulces.	lignes.	liures.	onces.	gros	grains
1	VT		1	0	0	0		10		4	0	0	2	0	0	0
		ton mi.														
2	RE		1	4	15	54		9		3	7	$2\frac{2}{5}$	1	12	12	58
		ton mai.														
3	MI		1	10	9	0		8		3	2	$4\frac{4}{5}$	1	9	9	43
		sem.mai.														
4	FA		1	14	3	32		$7\frac{1}{2}$		3	0	0	1	8	0	0
		ton mai.														
5	SOL		2	6	4	0		$6\frac{2}{3}$		2	8	0	1	5	5	24
		ton mi.														
6	RE		2	14	3	32		6		2	4	$9\frac{1}{5}$	1	3	3	14
		ton mai.														
7	MI		3	11	12	18		$5\frac{1}{3}$		2	1	$7\frac{1}{5}$	1	1	1	5
		semi.maj.														
8	FA		4	4	0	0		5		2	0	0	1	0	0	0

Figure 11: Mersenne's table, Harmonie, III 125–6

Another remarkable table by Mersenne. This time he provides data so that even a deaf may tune a string instrument, based on the tension, thickness, and length of its strings. The first table gives the tension of strings with equal length and thickness. The second table gives their thickness, measured as a diameter, with equal length and tension. The third table gives the length of strings with equal thickness and tension. The last table gives their tension when both thickness and length are variable. Alas, the fourth table is inconsistent with the second because in the second by thickness Mersenne means diameter, whereas the fourth makes sense only if thickness means cross-section. Such are the perils of tuning instruments by numerical ratios alone.

tances fallen by the spheres based on the different combinations of light and heavy elements. After him several others, such as Boyle and Newton, had recourse to numerical tables to study the properties of compressed air and the resistance of fluid to motion, for example.

It may be useful to attempt a classification of the tables we have seen thus far, bearing in mind that the same table may fit a variety of purposes and be used by readers in different ways. The first type presents empirical data unrelated to theory. For example, weights of various materials as those provided by Ghetaldi could not be predicted and calculated on the basis of any theory, but could only be measured empirically, compared, and tabulated. The aim of such tables appears to have been largely practical and utilitarian.

The second includes observational or experimental data with theoretical implications. Galileo's analysis of the positions of the 1572 nova falls into this category. Here tables were used in various forms to present data in a suitable form for drawing broader conclusions.

The third consists in calculation aids largely relying on trigonometry. The numbers rely on theory and the table works in a way not too dissimilar from a modern computer. Galileo's tables for shooting in day four of the *Discorsi* and Torricelli's in *De motu* fall into this category. This was the most common form for astronomical tables.

The last type seems to have a didactic, philosophical, and esthetic purpose at the same time. It presents tables obtained from elementary theoretical calculations bound within loosely empirical or meaningful limits, such as the maximum height whence a body can fall or the weight of the earth. Some tables show the natural numbers with their squares, for example, or the differences between successive squares, giving the progression of odd numbers. Here the purpose is not to ease computation, but rather to highlight the symmetry and regularity of certain phenomena, such as falling bodies, for example, or to find a height from the time of fall, or to ponder on the regularities of nature. It is not surprising that such tables figure quite prominently in a work titled *Harmonie universelle* and form a characteristic feature of Mersenne's worldview.

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