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Inherent and Centrifugal Forces in Newton

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Abstract

Over the last few years a resurgence of Newtonian studies has led to a deeper understanding of several aspects of his *Philosophiae naturalis principia mathematica*. Besides the new translation of Newton's masterpiece, these contributions touched on his mathematical style, investigative method, experimental endeavors, and conceptual systematization of key notions in mechanics and the science of motion.[2] With regard to the last topic, recent works have identified two notions where Newton's choices look unclear and scholarly opinion is divided. These notions are *materiae vis insita* or *vis inertiae*, namely the inherent force of matter or force of inertia, and *vis centrifuga* or centrifugal force. It is my conviction that the two notions became inter-related in Newton's thought starting from the time of composition of the *Principia* and that a new look at them will simultaneously clarify matters about both. Newton's beliefs about the nature of centrifugal force did not affect his calculations of planetary and cometary orbits in the *Principia*, but they are none the less of considerable intellectual interest.

After a brief introduction on Huygens and the early Newton, Sect. 2 focuses on Newton's views in the *Principia* and its preliminary manuscripts. Section 3 presents a summarized account of Leibniz's views, which are necessary for understanding Newton's criticism of them. This is the subject of Sect. 4, where we find his views most fully spelled out.

1. Huygens' centrifugal force and Newton ca. 1680

Let us consider a brief pre-history of the matter. Outward tendencies due to the circular motion of a body had long been taken into account by several scholars, notably

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[2] I. Newton, *The 'Principia'. A new translation by I. Bernard Cohen and Anne Whitman assisted by Julia Budenz* (Berkeley: University of California Press, 1999), hereafter *New translation*. Readers can find a useful bibliography on Newton in I.B. Cohen and G. Smith, eds, *The Cambridge Companion to Newton* (Cambridge: Cambridge University Press, 2002, hereafter *Cambridge Companion*), 465–80.

Descartes, but Christiaan Huygens was the first to provide in print a widely accepted way of measuring them and to coin the expression *vis centrifuga* in a set of propositions without proof in his *Horologium oscillatorium* of 1673. Huygens had already explored the matter in *De vi centrifuga* of 1659, a work first published in his 1703 *Opera posthuma*, and was to do it again in print in his *Discours de la cause de la pesanteur* of 1690. There is a conceptual dichotomy in the way Huygens introduced and applied the notion of centrifugal force, but there is no indication that it would have been perceived as troubling in the late 17th or the early 18th century. In *De vi centrifuga* he introduced centrifugal force as the tension in a string to which a weight is attached, where the other side of the string is held by an observer fixed on the rim of a rotating wheel. In this sense Huygensian centrifugal force has been aptly called a "static force", meaning a force that does not produce motion, but only tension in the string.[3]

Moreover, we should bear in mind that Huygens did not draw a distinction between what we call real and fictitious forces. For him the idea of an active spectator rotating at the center of a spinning wheel was partly a heuristic, partly a rhetorical device to tackle the issue. Like all scholars contemporary to him or of the following generation, Huygens took centrifugal force to be a real force due to the rotation of a body, not to the rotation of a frame of reference, as was normally done in much later formulations of classical mechanics. Huygens repeatedly compared centrifugal force to gravity by means of the rotation of a conical pendulum. Indeed, gravity itself, according to Huygens, was ultimately due to the centrifugal force of a rotating fluid vortex. Here we encounter the other side of centrifugal force, namely a force producing accelerated motion such as the acceleration of falling bodies on the earth's surface. It is in this sense that centrifugal force appears especially in *Discours de la cause de la pesanteur*, published in 1690 in the wake of Newton's 1687 *Principia* but actually largely drafted and presented to the Paris *Académie* in 1669. The table of contents of the treatise lists sections on "Centrifugal force compared to that of gravity" and "How it [centrifugal force] can be used to cause gravity." The mechanism envisaged by Huygens involved a fluid vortex rotating at such a high speed that all bodies on the earth are pushed toward its center because their centrifugal force is smaller than that of an equivalent volume of the vortex. Thus gravity results from a difference of centrifugal forces, and in this sense centrifugal force does produce motion, i.e., every time a heavy body falls.[4]

Newton had started investigating outward tendencies in curvilinear motion in his early manuscripts on motion. His earliest usage of the term "centrifugal force" must

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[3] C. Huygens, *Oeuvres Complètes* (The Hague, 1888–1950, hereafter *HOC*), 22 vols; 16, 253–301. G. Smith, "The methodology of the *Principia*", *Cambridge Companion*, 138–73, at 148. As to Descartes, the main published source known to contemporaries would have been *Principia philosophiae* (Amsterdam, 1644). See especially Part II, paragraph 39, and Part III, paragraphs 57–60.

[4] *HOC*, 21, 448: "La force Centrifuge comparée á celle de la Pesanteur"; "Comment elle peut servir à causer la Pesanteur". At 455 Huygens addressed the problem that gravity is directed toward a center of the earth rather than its axis of rotation by arguing that the subtle matter causing gravity moves around the earth in all directions.

surely be in his 1673 response to Huygens' gift of *Horologium oscillatorium*.[5] Around 1680 we find several references to *vis centrifuga* by Newton. Whereas Huygens was mainly concerned with gravity on the earth's surface, Newton's passages concern orbiting bodies. Although in both of the cases we are going to consider Newton did not specify his views about the cause of attraction, there is little doubt that in those years he believed it to be generated by some fluid, though through a mechanism different from Huygens'.[6] In both cases Newton clearly indicated his belief that orbital motion resulted from an imbalance between an attraction and centrifugal force originating from the body's curvilinear motion. In a 1679 letter to Hooke Newton discussed the case of the trajectory of a body allowed to move freely inside the earth toward its center, claiming that it will "circulate with an alternate ascent & descent made by it's *vis centrifuga* & gravity alternately overballancing one another." The words "alternately overbal[l]ancing" point to an oscillation resulting from two actions, where one is at times greater and at times smaller than the other. In a draft of a letter of 1681 probably intended to James Crompton for John Flamsteed, Newton argued that a comet would "fetch a compass about the sun", at perihelion its " *vis centrifuga*... overpow' ring the attraction & forcing the Comet there notwithstanding the attraction, to begin to recede from the sun."[7]

There is some disagreement among historians about the interpretation of Newton's texts and general views on centrifugal force. Most commentators have interpreted those passages as meaning that gravity or attraction and centrifugal force operate continuously throughout the orbit.[8] Recently it has been argued that Newton applied centrifugal

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[5] *The correspondence of Isaac Newton*, ed. by H.W. Turnbull, J.F. Scott, A.R. Hall, and L. Tilling (Cambridge, 1959–77, hereafter *NC*), 7 vols, 1, 290, Newton to Oldenburg, 23 June 1673. Some of Newton's early passages on curvilinear motion can be found in the *Waste Book*, the *Vellum Manuscript*, and *On Circular Motion*, in J. Herivel, *The background to Newton's "Principia"* (Oxford, 1965, hereafter *Background*), esp. 129–32, 145–7, 183–91, 192–8. For a study of the topic see D. Bertoloni Meli, "The Relativization of Centrifugal Force", *Isis*, 81, 1990, 23–43, hereafter *Relativization*.

[6] See for example Newton's letter to Boyle 28 February 1678/9 in *NC*, 2, 288–96, at 295 on the cause of gravity. Newton's conjecture relied ultimately on the varied "subtilty" of different particles of the aether. Z. Bechler, in "Newton's ontology of the force of inertia", in A. Shapiro and P. Harman, eds, *The Investigation of Difficult Things*, (Cambridge, 1992), 287–304, at 297, argued that "In a curved motion which is not the result of a mechanical push, there cannot be any centrifugal force exerted." His conclusion, however, seems unwarranted, since Newton did introduce centrifugal forces even after having rejected a mechanical cause for gravity. See also his *Newton's physics and the conceptual structure of the Scientific Revolution* (Dordrecht, 1991), ch. 11 and p. 302.

[7] These famous and often quoted passages are in *NC*, 2, 307–8, Newton to Hooke, 13 December 1679, at 307, and 358–62, Newton to [Crompton?], ca. April 1681, at 361. To these passages one may add Newton's reference to Giovanni Alfonso Borelli, *Theoricae Mediceorum Planetarum* (Florence, 1666), which invoked an imbalance between opposing tendencies in his study of orbital motion. See *NC*, 2, Newton to Halley, 435–41, at 437–8.

[8] For relevant comments see for example E.J. Aiton, *The vortex theory of planetary motion* (London-New York, 1972), 103; R.S. Westfall, *Force in Newton's physics* (London-New York, 1971), 426–7; D.T. Whiteside, ed., *The mathematical papers of Isaac Newton* (Cambridge, 1967–81, hereafter *NMW*), 8 vols, 6, 11n.32, talking of a "perpetual imbalance". Bertoloni Meli, *Relativization*, 33.

force only either to circular motion or to the points of an arbitrary curve where the attraction has the same direction as the radius of curvature.[9] This opinion, however, is highly problematic and is contradicted by direct textual evidence, as we are going to see below.[10]

Here I wish to emphasize one obvious yet important point of agreement, namely that in orbital motion centrifugal force – wherever it was defined - and gravity or attraction were not necessarily equal and opposite for Newton, but one could be larger, equal or smaller than the other. As we are going to see, his views on this matter were about to change.

2. *Principia mathematica* and its preliminary manuscripts

Newton's inclusion of a *vis insita* or *vis inertiae* in Definition 3 of the *Principia* has rightly seemed peculiar to several scholars, since a body's tendency to preserve its state of rest or uniform motion in a straight line hardly seems to require a force. I.B. Cohen's *Guide* to the *Principia* calls it "in many ways, the most puzzling of all the definitions in

[9] J.B. Brackenridge and M. Nauenberg, In "Curvature in Newton's dynamics", 88–9: "It is critical to note that for general orbital motion Newton never applied the term *centrifugal force* except when the radius is either a maximum or minimum, that is, at extreme points where the force does lie along the radius of the circle of curvature." According to their account, "Newton always applies the concept of centrifugal force only to circular motion, or to the maximum and minimum points for general orbital motion (where the force lies along the radius of the circle of curvature)", footnote 49 on 133–4, at 134. See also 95–106, where they analyze Newton's letter to Hooke quoted in the text. They leave no doubt about the hypothetical nature of their reconstruction: "We now turn to the consideration of the computational method employing curvature by which Newton *could have obtained* the curve" (100); "Thus, Newton *could have applied* his curvature method to determine the relation between motion on a given curve and the radial dependence of the force" (105-6) (my emphases). Nauenberg had outlined a similar analysis in "Newton's early computational method for dynamics", *Archive for History of Exact Sciences*, 46, 1994, 212–52, at 231. The reconstruction of Newton's calculations in his letters to Hooke relies on the idea of approximating the curve by means of a series of small circular arcs where each arc has a curvature appropriate to its corresponding portion of the curve. Newton had hinted at using curvature in a passage from the *Waste Book*, Herivel, *Background*, 130. The best transcription is in J.B. Brackenridge, *The key to Newton's dynamics* (Berkeley, 1995), 63: "If the body *b* moved in an Ellipsis then its force in each point (if its motion in that point bee given) may bee found by a tangent circle of Equall crookednesse with that point of the Ellipsis."

[10] The idea that Newton would apply centrifugal forces to one point of an orbit and not apply them to its neighboring points looks implausible. As I have argued in "Relativization", late 17th- and early 18th-century views on centrifugal force can be quite surprising to the modern reader and are not easy to determine in the absence of direct textual evidence. The views of Brackenridge and Nauenberg, however, rely on their own dynamical interpretation of their own mathematical reconstruction of Newton's calculations associated with his correspondence with Hooke. They then generalize their hypothetical interpretation to cover Newton's later views as well without weighing the available documentary evidence.

the *Principia*."[11]. In order to start unraveling the puzzle, let's look at the statement of the definition:[12]

> Definition 3. Inherent force of matter is the power of resisting by which every body, so far as it is able, perseveres in its state either of resting or of moving uniformly straight forward.
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> Definition 3. Materiae vis insita est potentia resistendi, qua corpus unumquodque, quatenum in se est, perseverat in statu suo vel quiescendi vel movendi uniformiter in directum.

Hence the inherent force of matter would seem not to be a force at all. The explanation of this definition introduces the term *vis inertiae* as a synonym for the inherent force of matter:[13]

> This force is always proportional to the body and does not differ in any way from the inertia of the mass except in the manner in which it is conceived. Because of the inertia of

[11] *New translation*, 96. See also 96–101. For centrifugal force see also 82–4, reporting the views of William Whiston and John Harris. I. Bernard Cohen, "Newton's concepts of force and mass, with notes on the Laws of Motion", in *Cambridge Companion*, 57–84, at 60–2, provides a useful historical study of *vis insita* and *vis inertiae*. A. Gabbey, "Force and inertia in seventeenth-century dynamics", *Studies in History and Philosophy of Science*, 2, 1971, 1–67, especially section III, 31–50, also in an expanded version, "Force and inertia in the seventeenth century: Descartes and Newton", in S. Gaukroger, ed., *Descartes. Philosophy, mathematics, and physics* (Sussex, 1980), 230–320, at 272–86.

[12] *New translation*, 404. *Third edition with variant readings*, 2. Unless otherwise specified, the text remained substantially unchanged over the three editions. Newton had expressed analogous views in several earlier manuscripts, such as the Waste Book, in Herivel, *Background*, 157: "A body is saide to have more or lesse motion as it is moved with more or lesse force, that is as there is more or lesse force required to generate or destroy its whole motion." *The Laws of Motion*, in Herivel, *Background*, 208: "But the motion it selfe and the force to persevere in that motion is more or lesse accordingly as the factus of the bodys bulk into its velocity is more or lesse. And that force is equivalent to that motion which it is able to beget or destroy." See also *De gravitatione et aequipondio fluidorum*, in A.R. Hall and M.B. Hall, eds, *Unpublished scientific papers of Isaac Newton* (Cambridge, 1962), 114 and 148: "Def 5. Vis est motus et quietis causale principium. Estque vel externum quod in aliquod corpus impressum motum ejus vel generat vel destruit, vel aliquo saltem modo mutat, vel est internum principium quo motus vel quies corpori indita conservatur, et quodlibet ens in suo statu perseverare conatur & impeditum reluctatur." "Definition 5. Force is the casual principle of motion and rest. And it is either an external one that generates or destroys or otherwise changes impressed motion in some body; or it is an internal principle by which existing motion or rest is conserved in a body, and by which any being endeavours to continue in its state and opposes resistance."

[13] *New translation*, 404. *Third edition with variant readings*, 2. Compare also *De gravitatione et aequipondio fluidorum*, in A.R. Hall and M.B. Hall, eds, *Unpublished scientific papers of Isaac Newton* (Cambridge, 1962), 114 and 148: "Def 8. Inertia est vis interna corporis ne status ejus externa vi illata facile mutetur." "Definition 8. Inertia is force within the body, lest its state should be easily changed by an external exciting force." It is also worth comparing Descartes, *Principia*, Part II, paragraph 43, "In quo consistat vis cujusque corporis ad agendum vel resistendum". I quote from the contemporary French translation, *Les principes de la philosophie* (Paris, 1647),

> matter, every body is only with difficulty put out of its state either of resting or of moving. Consequently, inherent force may also be called by the very significant name of force of inertia.

> Haec semper proportionalis est suo corpori, neque differt quicquam ab inertia massae, nisi in modo concipiendi. Per inertiam materiae fit ut corpus omne de statu suo vel quiescendi vel movendi difficulter deturbetur. Unde etiam vis insita nomine significantissimo vis inertiae dici possit.

So far the issue seems to be largely linguistic. But with the reference to the difficulty of changing a body's state, Newton is preparing the ground for his subsequent important qualification about the circumstances of when this force is exerted:[14]

> Moreover, a body exerts this force only during a change of its state, caused by another force impressed upon it$_{[.]}$

> Excercet vero corpus hanc vim solummodo in mutatione status sui per vim aliam in se impressam facta$_{[.]}$

Thus the force of inertia should be seen as a force present in the body at all times, but actually exerted only during a change of state. For this reason John Herivel perceptively called it a "potential" force. For the same reasons I shall call it a latent force ready to manifest itself only in the right circumstances. Thus no impressed force is required for a body to preserve its state of rest or rectilinear uniform motion; or, as Newton put it, impressed force "consists solely in the action and does not remain in a body after the action has ceased. For a body perseveres in any new state solely by the force of inertia." Impressed force, following Definition 4, "is the action exerted on a body to change its state." The examples of impressed force provided by Newton include "percussion, pressure, or centripetal force."[15]

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which is slightly more explicit: "lors qu'il [a body] est en repos, il a de la force pour demeurer en ce repos & pour resister à tout ce qui pourroit le faire changer. De mesme que, lors qu'il se meut, il a de la force pour continuer de se mouvoir avec la mesme vitesse & vers le mesme costé." In considering the quantity of this force Descartes includes factors such as the size of the body and its speed. M. Gueroult, "The metaphysics of force in Descartes", in S. Gaukroger, ed., *Descartes* (Sussex, 1980), 196–229; Gabbey, "Force and inertia", 23–6 (267–9 of the expanded version); D. Garber, *Descartes' metaphysical physics* (Chicago, 1992), 293–9.

[14] *New translation*, 404. *Third edition with variant readings*, 2.

[15] *New translation*, 405. Therefore impressed force does not have the same physical dimensions all the times, since for percussion one would consider quantity of motion and for centripetal force the change of quantity of motion in a given time. Herivel, *Background*, 28. In this context I take the terms "latent" and "potential" as equivalent. Gabbey, "Force and inertia", 40–2 (278–9 of the expanded version), prefers to argue that in the *Principia* Newton employs the notion of *vis inertiae* in two senses, as a resisting force equal and opposite to the *vis impressa*, and as a maintaining force measured by the body's quantity of motion. Herivel, ibidem, and especially Gabbey, at 33–5 (273–5 of the expanded version), draw attention to Newton's usage of the term "solummodo" or "only" in Definition 3 (see quotation in the text).

Newton proceeds further to qualify his statement by introducing a twofold perspective from which a body exerting its force of inertia can be viewed:[16]

> [T]his exercise of force is, depending on the viewpoint, both resistance and impetus: resistance insofar as the body, in order to maintain its state, strives against the impressed force, and impetus insofar as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavors to change the state of that obstacle.

> [E]stque exercitium illud sub diverso respectu & resistentia & impetus: resistentia, quatenus corpus ad conservandum statum suum reluctatur vi impressae; impetus, quatenus corpus idem, vi resistentis obstaculi difficulter cedendo, conatur statum ejus mutare.

Newton then proceeds to challenge the commonly held view that resistance should pertain to resting bodies and impetus to moving bodies, arguing that bodies commonly considered at rest may not be truly so.

Newton's twofold perspective requires explanation. Seen from the standpoint of the body striving against the impressed force, or reacting to it, the force of inertia manifests itself as resistance. However, seen from the perspective of the body striving against a resisting obstacle, the force of inertia manifests itself as an endeavor to change the state of that obstacle. One gets the impression that resistance, when the body strives against the impressed force, is seen from the perspective of the body, whereas impetus concerns the body's action against another body. Newton's text is somewhat cryptic, and interpretations must remain correspondingly tentative. Fortunately the preliminary manuscripts to the *Principia* shed some light on the matter.

It is well known that in the autumn of 1684 Newton drafted *De motu corporum in gyrum*, the first step on the road to his *Principia*. In a set of revised definitions and laws under the heading *De motu corporum in mediis regulariter cedentibus*, dated by Whiteside to the winter of 1684/5, Newton struggled with the notions of *vis insita*, *vis exercita*, and *vis centrifuga*. The paleographic strata are quite complex and require careful handling; therefore I shall not rely on current transcriptions and translations, but rather go back to the manuscripts in the wonderful photographic reproductions edited by Whiteside. Although reconstructing the sequence of Newton's additions and cancellations in its entirety seems arduous, some steps can be retraced with a good degree of certainty.[17]

Initially Newton had Definition 12 for *Corporis vis insita*, Definition 13 for *Vis motus*, and Definition 14 for *Vis corporis illata et impressa*. Later he added a new Definition 14 for *Corporis vis exercita*, and renumbered subsequent definitions accordingly. The first version of Definitions 12 and 13 reads:[18]

[16] *New translation*, 404. *Third edition with variant readings*, 2.

[17] D.T. Whiteside, ed., *The preliminary manuscripts for Isaac Newton's 'Principia'. 1684–1686* (Cambridge, 1989), hereafter *Preliminary manuscripts*.

[18] Whiteside, *Preliminary manuscripts*, 30–31. Herivel, *Background*, 306 and 311. *NMW*, 6, 191.

Def. 12. The inherent, innate, and essential force of a body is the power by which it perseveres in its state either of resting or of moving uniformly in a straight line, and is proportional to the quantity of the body.

Def. 13. The force of motion or [force] adventitious to the body from its own motion is that whereby the body strives to conserve its entire quantity of motion. It is commonly called impetus and is proportional to the motion, and according to the type of motion is absolute or relative. ~~The centrifugal force of orbiting bodies is to be ascribed to the absolute kind.~~

Def. 12. Corporis vis insita innata et essentialis est potentia qua id perseverat in statu suo quiescendi vel movendi uniformiter in linea recta, estque corporis quantitati proportionalis.

Def. 13. Vis motus seu corpori ex motu suo adventitia est qua corpus quantitatem totam sui motus conservare conatur. Ea vulgo dicitur impetus estque motui proportionalis, et pro genere motus vel absoluta est vel relativa. ~~Ad absolutam referenda est vis centrifuga gyrantium.~~

After having defined inherent forces, in Definition 13 Newton introduced the notion of centrifugal force as an example of *vis motus absoluta*. Since in this set of definitions and laws Newton introduced the notions of absolute time, space, and motion, it is possible that he wanted to denote centrifugal force as absolute because of its association with circular motion. In Definition 9 he had stated that absolute and relative motions can be distinguished by the *conatus recedendi a centro*.[19]

Subsequently Newton deleted the last sentence of Definition 13, as shown in my transcription, and added a new Definition 14 for *Corporis vis exercita*:[20]

Def. 14. The exerted force of a body is that by which it strives to preserve that part of its state either of resting or of moving that it loses at individual moments, and it is proportional to the change of that state or to that part lost at individual moments, and not improperly it is called reluctance or resistance of the body. One species of this is the centrifugal force of orbiting bodies.

Def. 14. Corporis vis exercita est qua id conatur conservare status sui movendi vel quiescendi partem illam quam singulis momentis amittit, estque status illius mutationi seu parti singulis momentis amissae proportionalis, nec improprie reluctatio vel resistentia corporis dicitur. Hujus una species est vis centrifuga gyrantium.

Here Newton introduces the notion that *vis exercita* is proportional to the body's change of state, and centrifugal force is a type of *vis exercita*. Thus both Definitions 13, in its original form, and 14 ended with an attempt to account for centrifugal force.

At some later stage Newton deleted both Definitions 13 and 14 in their entirety, adding the contents of Definition 14 to the bottom of Definition 12. Thus despite the cancellation, the contents of Definition 14 were largely preserved by Newton. In Definition

[19] Whiteside, *Preliminary manuscripts*, 31. Similar arguments can be found in the *Principia*, scholium to the definitions, *New translation*, 408–15.

[20] Whiteside, *Preliminary manuscripts*, 30. Herivel, *Background*, 317 and 320.

12 he changed the period after "proportionalis" into a comma and added the following text:[21]

> moreover, it is exerted proportionally to the change of state and in so far as it is exerted it can be called the exerted force, endeavor, and reluctance of a body. One species of this is the centrifugal force of orbiting bodies.
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> exercetur vero proportionaliter mutationi status et quatenus exercetur dici potest corporis vis exercita, conatus et reluctatio. Hujus una species est vis centrifuga gyrantium.

The stratification in Newton's text is quite daunting and I have reconstructed it only in part. Partial as this reconstruction is, however, it does provide us with an example of *vis insita* from the time of composition of the *Principia*, namely the centrifugal force of orbiting bodies. Newton seems to imply that the centrifugal force of an orbiting body is proportional to the change of its state. Westfall was entirely right in perceiving here a link with the third law of motion, expressing the equality of action and reaction. Westfall states that "the idea of exerted force was a groping step toward the insight expressed in the third law, first announced later in this same paper. If a body resists the actions seeking to change its state of motion, it exerts in reaction a force on whatever acts on it." Indeed, an addition to the version of the third law found later in the same manuscript provides powerful textual evidence for this link. I give first the original enunciation of the law:[22]

> Law 3. Any body suffers a reaction as much as it acts on another. Whatever presses or pulls is pressed or pulled by it in the same amount. If a bladder full of air presses or carries another similar to itself, both yield equally inwards.[23] If a body impinging on another changes by its force the motion of the other, its own motion too (by the equality of the mutual pressures) will be changed by the force of the other. If a magnet attracts iron, it too is equally attracted in turn, and likewise in other cases.
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> Lex 3. Corpus omne tantus pati reactione quantum agit in alterum. Quicquid premit vel trahit alterum, ab eo tantum premitur vel trahitur. Si vesica aere plena premit vel ferit alteram sibi consimilem cedet utraque aequaliter introrsum. Si corpus impingens in alterum vi sua mutat motum alterius et ipsius motus (ob aequalitatem pressionis mutuae) vi

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[21] Whiteside, *Preliminary manuscripts*, 31. Herivel, *Background*, 306 and 311. See also Westfall, *Force*, 449. For the translation of "gyrantium" I rely on Whiteside's note to his translation of "in gyrum", *NMW*, 6, 30-1n.3: "Literally a closed circuit, but understand any path which is everywhere convex round some internal point." Gabbey, "Force and inertia", 34 (extended version, 273), omits the last portion of the extended Definition 12, with the passage on centrifugal force. He discusses centrifugal force only on 66–7, but interestingly the relevant passage is expunged from the expanded version, at 296.

[22] Westfall, *Force*, 451. Preliminary versions of the third law can be found in the early studies on collision in the *Waste Book*, dating from the 1660s, Herivel, *Background*, 159, and in the *Lectures on Algebra*, *NMW*, 6, 148–9, dated 1675. See also Gabbey, "Force and inertia", 35–6, 39, 46–7 (extended version, 274–5, 277, 283). Whiteside, *Preliminary manuscripts*, 32. *NMW*, 6, 193. Herivel, *Background*, 307 and 312–3. I have slightly modified Herivel's translation.

[23] This appears to be a reference to Mariotte's essay, *Traitté de la percussion* (Paris, 1673), quoted in *Principia*, *New translation*, 425.

alterius tantum mutabitur. Si magnes trahit ferrum ipse vicissim tantum trahitur, et sic in alijs.

After this enunciation, Newton added a justification for the third law relying on the previous definitions. The numerous cancellations mean that the numbering is in disarray and the identification problematic, but the references to *vis exercita* and *vis impressa* indicate that Newton presumably intended those definitions:[24]

Moreover this Law is established from Definitions 12 and 14, in that the exerted force of a body to conserve its state is the same as the impressed force in the other body to change its state, and the change of state of the first is proportional to the first force, that of the second to the second.

Constat vero haec Lex per Def. 12 et 14 in quantum vis corporis ad status sui conservationem exercita sit eadem cum vi in corpus alterum ad illius statum mutandum impressa, et vi priori proportionalis sit mutatio status priori posteriori ea posterioris.

Here the third law of motion is made dependent on the previous definitions. Through the notion of *vis exercita*, the third law becomes conceptually tied to *vis centrifuga*.

Newton's ideas at this stage can be summarized as follows. *Vis insita* is always present in a body as a latent force, but it manifests itself only when the body changes its state of rest or rectilinear uniform motion. The case of impact is quite straightforward and I will not discuss it here. The case of magnetism, however, is intriguing because it suggests that two bodies act upon each other with equal and opposite forces even for attractions. When an orbiting body comes under the influence of a centripetal force, its reaction can be seen from two perspectives both governed by the third law of motion. First, the orbiting body acts on the source of the centripetal force with a force equal and opposite to that exerted on itself. If body A attracts body B with a given force, B also attracts A with an equal and opposite force.[25] Second, if we consider the body in itself and its resistance as a reaction to the action seeking to divert it from its own path, the body exerts a centrifugal force equal and opposite to the centripetal force. Thus in the preliminary manuscripts to the *Principia* Newton understood centrifugal force for an orbiting body as the body's reaction to centripetal force. This double perspective may be a reason why Newton did not consider centrifugal forces in his computations for orbital motion: they were another manifestation of a force that had already been taken into account by the reciprocity of attraction. Although Newton did not specify whether he was referring to orbital trajectories in general or simply to circular ones, the general tone of his analysis suggests that he was referring to the general case, not simply to

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[24] Whiteside, *Preliminary manuscripts*, 32. *NMW*, 6, 193, where in my opinion Whiteside renumbered the propositions incorrectly. Herivel, *Background*, 307 and 313. I have slightly modified Herivel's translation. The definition of *vis impressa*, 15 in the final numbering, is: "Def. 15. Vis corporis illata et impressa est qua corpus urgetur mutare statum suum movendi vel quiescendi", followed by the addition: "estque diversarum specierum, sicut pulsus seu pressio percutientis, pressio continua, vis centripeta, resistentia medij."

[25] This framework is made more explicit in the published *Principia* in the scholium to the laws of motion, *New translation*, 424–30, where Newton argued that the law is valid also for attractions. See also Proposition 69, Book I, 587-8. Bertoloni Meli, *Relativization*, 32.

circular motion. In no passage I am aware of did Newton suggest that centrifugal force applies only to circular motion or to the case when the attraction has the same direction as the radius of curvature.

Moving to the *Principia*, one is left with some doubts as to Newton's views, because he did not provide a clear explanation of what he meant by centrifugal force and of how it had to be calculated when the orbit is not circular but eccentric. In a passage following Definition 5 of centripetal force Newton revisits a Cartesian image:[26]

> A stone whirled in a sling endeavors to leave the hand that is whirling it, and by its endeavor it stretches the sling, doing so the more strongly, the more swiftly it revolves; and as soon as it is released, it flies away. *The force opposed to that endeavor*, that is, the force by which the sling continually draws the stone back toward the hand and keeps it in an orbit, I call centripetal, since it is directed toward the hand as toward the center of an orbit. And the same applies to all bodies that are made to move in orbits.

> Lapis, in funda circumactus, a circumagente manu abire conatur; & conatu suo fundam distendit, eoque fortius quo celerius revolvitur; & quamprimum demittitur, avolat. *Vim conatui illi contrariam*, qua funda lapidem in manum perpetuo retrahit & in orbe retinet, quoniam in manu ceu orbis centrum dirigitur, centripetam appello. Et par est ratio corporum omnium, quae in gyrum aguntur.

Notice that the passage italicized, in Latin "Vim conatui illam contrariam", suggests that Newton had the third law in mind. The last sentence offers a generalization indicating that Newton may have had non-circular orbits in mind, though he is not absolutely explicit about this.

In the first two books Newton provided a wealth of mathematical results often involving centripetal forces and resistance forces depending on an arbitrary power of the distance and velocity, respectively.[27] At times Newton also considered negative centripetal forces and called them centrifugal, but these were just repulsive forces that did not have a conceptual affinity with the centrifugal force that concerns us here.[28] He also considered centrifugal forces on a rotating earth, but these depend on the earth's speed of rotation and radius at the appropriate latitude and are not equal and opposite to centripetal forces, unlike the case of orbiting bodies, because a body on the earth's surface is kept in place largely by the earth's reaction to the body's gravitational weight.[29] An

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[26] *New translation*, 405. *Third edition with variant readings*, 45. This passage is not present in the first edition. Descartes, *Principia philosophiae*, II, 39.

[27] This point is developed in G. Smith, "The methodology in the *Principia*", in *Cambridge Companion*, 138–73.

[28] Among these instances is Corollary 3 to proposition 41, Book I, where Newton seeks the trajectory of an orbiting body given an arbitrary centripetal force, and Proposition 23, Book II, where he considers the case of a fluid composed of particles repelling each other. *New translation*, 531–2 and 697–9.

[29] See for example Book III, Proposition 19, *New translation*, 821–6. In orbital motion centripetal force is counterbalanced by a centrifugal force. The gravitational weight of a body on the earth is counterbalanced by the impenetrability of the earth's surface and the body's centrifugal force, but the former is preponderant over the latter; therefore centrifugal force merely reduces the body's weight.

interesting exception occurs in the Scholium to Proposition 4, Book I, where Newton discussed the motion of a body rotating inside a hollow cylinder or circle. By considering the rotation as the limit case of an infinite number of impacts, Newton was able to provide a quantitative measure of the body's force on the circle, concluding:[30]

> This is the centrifugal force with which the body urges the circle; and *the opposite force*, with which the circle continually repels the body toward the center, *is equal to this centrifugal force*.
>
> Haec est vis centrifuga, qua corpus urget circulum; *& huic aequalis est vis contraria*, qua circulus continuo repellit corpus centrum versus.

The case discussed here is peculiar in that it involves a material constraint rather than an orbiting body attracted with a centripetal force. It is noteworthy that Newton's language includes an implicit reference to the third law of motion with the words I italicized, " *& huic aequalis est vis contraria*", where " *huic*" refers to " *vis centrifuga*".

In Book III Newton considered one relevant case. In the Scholium to Proposition 4 he discussed the motion of hypothetical moons rotating around the earth, on the example of the satellites of Jupiter and Saturn. If the lowest of these hypothetical moons were small and rotated very close to the top of high mountains, Newton claimed that, were it to be deprived of its motion, "as a result of the absence of the centrifugal force with which it had remained in its orbit", it would fall toward the earth like a heavy body.[31]

In conclusion, although the *Principia* does not provide an unambiguous account of Newton's views, nowhere does he suggest that centrifugal force is fictitious and originates from a choice of a reference frame. Rather, the evidence points to Newton having reinterpreted centrifugal force in terms of the third law of motion.[32] One may wonder, if he truly had the third law in mind, why he did not say so more explicitly. In the published *Principia* Newton deleted all reference to centrifugal force in the definition of *vis insita*. Part of the answer may reside in the logical structure of the work. Laws follow definitions both in the preliminary manuscripts and in the published *Principia*, so Newton could not have had recourse to a law at that early stage, allowing him to introduce centrifugal force as a reaction to a centripetal force based on the third law of motion, *before* having

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[30] *New translation*, 453. The expression *vis centrifuga* is not present in this passage in the first edition, but the gist of the passage remained substantially the same. Newton had examined a similar case in the mid-1660s in the *Waste Book* discussed in Herivel, Background, 7–13, 45–8, and 128–32. Significant corrections are in the essay review by D.T. Whiteside, "Newtonian dynamics", *History of Science*, 5, 1966, 104–17. In Part V of *Horologium Oscillatorium* (Paris, 1673), Theorem 6, Huygens analyzed the case of a body moving on the concave surface of a parabolic conoid. His case too involved a material constraint.

[31] *New translation*, 805, only in the third edition. Notice here with regard to the discussion in footnote 4 above that Newton applied centrifugal forces to orbits of satellites that are not all rigorously circular.

[32] As Eric Aiton put it in *Vortex theory*, 102: "The belief in the reality of the centrifugal force, however, betrays some conceptual confusion which could only be resolved by the application of the third law of motion to the forces between the two bodies." Aiton would agree, I believe, that Newton's attempted resolution of this conundrum was problematic.

formally defined either centripetal force or the third law itself. But Newton may also have thought that centrifugal force was sufficiently well known and did not figure in his actual calculations of orbital motion. To my knowledge, even at the height of the priority dispute, he was not attacked over his treatment of centrifugal force or *vis insita*.

In order to find more explicit statements, we will have to examine a manuscript against Leibniz drafted by Newton in the mid-1710s. Before examining it, however, we have briefly to discuss the views Newton was attacking.

3. Leibniz's views on orbital motion

Soon after the publication of the *Principia*, Leibniz published in the *Acta Eruditorum* for 1689 an essay on planetary motion, *Tentamen de motuum coelestium causis*, where he tried to recover some of Newton's results in a different fashion. Despite his protestation, the manuscript evidence shows that Leibniz relied on Newton's book and deftly transformed Newton's framework in the attempt to provide a mechanical explanation for orbital motion. Leibniz's work is quite interesting in its own right and worthy of careful reflection, but for the purposes of this paper a brief account will suffice.[33]

Newton decomposed orbital paths into a rectilinear uniform motion and an accelerated motion toward a center of force. By contrast, Leibniz decomposed orbital trajectories into a circular motion and a radial motion due to the imbalance between a centrifugal endeavor and the solicitation of gravity, or a centripetal tendency. The term endeavor or *conatus* suggests an action internal to the body, whereas the term solicitation or *solicitatio* suggests an external action. Indeed, according to Leibniz the former was internal to the body and was due to its circular motion, whereas the latter was due to the external action of a fluid vortex pressing the body toward the center. Leibniz called radial the motion toward or away from the center "paracentric". The circular motion was due to a vortex rotating with a speed inversely proportional to the distance from the center; thus the radii from the center to the orbiting body swept out equal areas in equal times. The endeavors in the radial motion were due to the body's circular motion and to the impulsion of the fluid vortex. Leibniz found the centripetal endeavor to be inversely as the square of the distance, whereas the centrifugal endeavor was as the square of the speed of rotation over the distance. The rotation speed was the circular component of orbital speed, or as we would say, the component perpendicular to the radius.

We now enter the peculiar world of Leibnizian mathematics and mechanics. Geometrically, centrifugal endeavor can be expressed as $r - r\cos \mathrm{d}\phi$, where r is the radius and ϕ is the angle of rotation or circulation. Since the angle is very small, Leibniz took its differential $\mathrm{d}\phi$. The segment $r - r\cos\phi$ is called the versed sine of the angle ϕ; therefore according to Leibniz centrifugal endeavor is represented by the versed sine of the differential of the angle of circulation. Since $\mathrm{d}\phi$ is very small, centrifugal endeavor is proportional to $r(\mathrm{d}\phi)^2$. Leibniz took the differential of time $\mathrm{d}t$ to be constant; therefore $r(\mathrm{d}\phi)^2$ is indeed proportional to the square of the velocity of rotation over the radius,

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[33] D. Bertoloni Meli, *Equivalence*, provides the most comprehensive account of Leibniz's views on planetary motion, especially chs 1-5; ch. 6 contains a translation of Leibniz's essay.

or $(r\mathrm{d}\phi/\mathrm{d}t)^2/r$. Since the velocity of rotation is inversely as the distance, centrifugal endeavor is inversely as r^3 . Leibniz's calculations apply in general to orbital motion, such as motion along an ellipse for example, not simply circular motion. Thus he found that the second order differential of the radius or, as he called it, the element of paracentric impetus, resulted from the imbalance between a centrifugal endeavor inversely proportional to the third power of the distance and a centripetal one inversely proportional to the square of the distance. We can present his result in a slightly modernized form as:

$$\mathrm{dd}r/\mathrm{d}t^2 = (\alpha/r^3) - (\beta/\mathrm{r}^2) \quad (1)$$

where α and β are appropriate constant factors.

4. Newton's attack and clarifications

Newton criticized many aspects of Leibniz's account, but one text is especially relevant to our discussion. It is a comprehensive attack contained in a folio sheet in his hand titled *Ex Epistola cujusdam ad Amicum*, probably dating from the mid-1710s. The manuscript was found among John Keill's papers and was published by Joseph Edleston in 1850. It is now preserved among the papers in the University Archives of Cambridge University Library.[34] The text contains quite extensive cancellations and reworking, but two passages crucial for our purposes contain very few emendations, namely short additions to the second passage. In the manuscript, after a brief attack on an essay by Leibniz on motion in a resisting medium, Newton provides a point-by-point criticism of Leibniz's *Tentamen de motuum coelestium causis*. The first relevant passage, referring to Newton in the third person, states:[35]

> The eleventh Proposition in the *Tentamen* is this: "The centrifugal endeavor can be expressed by the versed sine of the angle of circulation". This Proposition is certainly true whenever the circulation occurs in a circle without paracentric motion. But whenever it occurs in an eccentric orbit the Proposition is not true. The centrifugal endeavor is always

[34] Cambridge University Library (hereafter CUL), Classmark UA O.XIV.278.8 (xiii). I would like to thank Adam Perkins, Virginia Cox, and Stephen Lees for their assistance in identifying this manuscript and procuring a microfilm of it. J. Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes* (London, 1850, reprinted 1969), 307–114, where *Ex epistola* is seen as related to the *Commercium epistolicum*. The same manuscript is mentioned in *NC*, 6, 114-5n.5, where it is dated about 1714 by the editors. The significance of this manuscript was identified by Aiton, *Vortex theory*, 102–3, and Bertoloni Meli, "Relativization", 32. Brackenridge and Nauenberg do not mention it. *NC*, 6, 116–22 contains another attack on the *Tentamen* and is discussed in Bertoloni Meli, *Equivalence*, 189–90. Keill had asked for Newton's help in criticizing Leibniz's *Tentamen*, *NC*, 6, 113-4, Keill to Newton, Oxford, 2 May 1714.

[35] Cambridge University Library, UA O.XIV.278.8 (xiii), 1v. Edleston, 311. The essay on motion in resisting media by Leibniz is "Schediasma de resistentia medii", in *Acta Eruditorum*, Jan. 1689, 38–47.

> equal and opposite to the force of gravity for the third Law of motion in Newton's *Principia Mathematica*, and the force of gravity cannot be expressed by the versed sine of the angle of circulation, but is inversely as the square of the Radius.
>
> Undecima tentaminis Propositio est haec. *Conatus centrifugus exprimi potest per sinum versum anguli circulationis*. Et vera quidem est haec Propositio ubi circulatio fit in circulo sine motu paracentrico. Sed ubi fit in Orbe excentrico Propositio vera non est. Conatus centrifugus semper aequalis est vi gravitatis & in contrarias partes dirigitur per tertiam motus Legem in Principiis Mathematicis Newtoni, et vis gravitatis exprimi non potest per sinum versum anguli circulationis, sed est reciproce ut quadratum Radii.

After having quoted Leibniz's Proposition 11, Newton challenges its generality, arguing that it is valid only for a circular orbit, not for an eccentric one. The reason for this is the third law of motion in Newton's *Principia*, whereby centrifugal conatus and gravity are always equal and opposite. While commenting on Leibniz's text, Newton used the term *conatus* or endeavor, as used by Leibniz in the *Tentamen*. Since he reverted to the more common term "force" in the following passage, the two expressions have to be seen as equivalent. I fail to see in this quotation any sign that for Newton centrifugal force was restricted to only two points in the orbit, where centripetal force is directed in the direction of the center of curvature. On the contrary, the reference to a law of motion and the word *semper* clearly indicate that Newton's reasoning applies to the entire orbit.

In a later passage of the same manuscript Newton defended the same view again, this time with important terminological clarifications. I have indicated additions in angled brackets:[36]

> Propositions 21 and 25, maintaining that centrifugal force is smaller than the gravity of the planet towards the sun, are therefore false.[37] The orbital motion of the planet does not depend on the excess of gravity over centrifugal force (as Leibniz believes), but <the Orbit is curved> by <the action> of gravity alone, to which centrifugal force <(as a reaction or resistance)> is always equal and opposite on account of the third Law of motion put forward by Newton.
>
> Propositio vegesima {*sic*} prima et Propositio vigesima quinta, minorem exhibent vim centrifugam quam gravitatem Planetae in Solem ideoque falsae sunt. Motus Planetae in orbe non pendet ab excessu gravitatis supra vim centrifugam (uti credit Leibnitius) sed <Orbis incurvatur> a gravitatis <actione> sola, cui vis centrifuga <(ut reactio vel resistentia)> semper est aequalis & contraria per motus Legem tertiam a Newtono positam.

Here too Newton argues that a planet's orbit is bent by gravity alone, adding that centrifugal force is always equal and opposite to it on account of the third law of motion. Once again, I fail to detect any hint that Newton would consider centrifugal force in only two points. One of the additions is especially significant, namely where Newton

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36 CUL, UA O.XIV.278.8 (xiii), 2v. Edleston, 313.

37 Originally Leibniz had made a mistake by a factor of two in his determination of the centrifugal endeavor, which he found to be half of the correct value. In an article in the *Acta Eruditorum* for October 1706, 446–51, he corrected the mistake. Leibniz's error and correction are explained in Bertoloni Meli, *Equivalence*, 80–3.

states that centrifugal force is like a *reactio vel resistentia* to the action of gravity. The idea of centrifugal force being a resistance is one we have already encountered in the preliminary manuscripts to the *Principia* discussed in Sect. 2 above. The same notion of resistance was implicitly linked to centrifugal force in Definition 3 of *vis insita* in the *Principia*. The term *reactio* is linked to the third law invoked by Newton.

Hence the second passage of *Ex Epistola cujusdam ad Amicum* reveals a terminological and intellectual continuity in Newton's thought going back approximately thirty years to the time of the preliminary manuscripts to the *Principia*. This continuity involves the connection between *vis insita* and centrifugal force as a resistance or a body's reaction to a force attempting to deflect it from its rectilinear uniform motion. As such, centrifugal force in orbital motion is one possible manifestation of *vis insita* and is equal and opposite to centripetal force at each point in the orbit.

That later in the dispute with Leibniz the Oxford mathematician John Keill attacked Leibniz precisely over this point should come as no surprise, once we bear in mind the extensive contacts between Keill and Newton, and recall that Newton's manuscript *Ex Epistola cujusdam ad Amicum* was found among Keill's papers. In a letter to Newton on June 2, 1714, Keill proposed the following emendations for an article later published in French without substantive changes in the *Journal Literaire de la Haye*:[38]

> As Mr Leibnits is generally very obscure in his philosophical notions, so he seems here not to be very clear about the centrifugal force, which in reality is nothing but the Reaction to the Centripetal force: or the Resistance arising from the Vis Inertiae, that a body has to be turned out of its direction. And therefore the centripetal and centrifugal forces are always equal and contrary to each other.

All the main and most characteristic ingredients of Newton's interpretation of centrifugal force in orbital motion can be found in Keill's text, notably the recourse to the third law of motion and the usage of the term "reaction", as well as the reference to *vis inertiae* and the usage of the term "resistance". Keill merely put in print views Newton had mulled over for decades and adumbrated though not fully stated in several passages in the *Principia*.

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[38] *NC*, 6, 148–50, at 148–9. I have expanded contractions in the original text. J. Keill, "Réponse", *Journal Literaire de la Haye* for July and August 1714, 319–58, at 350-1, where he claims that centrifugal force is the effort a body makes to resist any changes to its state of rest or rectilinear uniform motion. A relevant later text is quoted in Bertoloni Meli, *Relativization*, 34. In "Curvature in Newton's dynamics", while arguing that Newton used the notion of centrifugal force either for circular motion or, if motion is not circular, exclusively where the radius is a maximum or a minimum, Brackenridge and Nauenberg seem to draw a distinction between Newton and Keill. While citing my claim from page 34 of *Relativization*, they seem to cast doubt on the identity of Keill's and Newton's views, at 131n.23: "John Keill (presumably representing Newton's position)". However, the textual evidence clearly supports the claim that Keill represented Newton's position.

5. Conclusion

From the mid-1680s onwards the key terms *reactio* or *vis contraria* and *resistentia* represent an intellectual bridge between the notions of *vis insita* and *vis centrifuga* in Newton's analysis of orbital motion. As we have seen in the preliminary manuscripts to the *Principia* and in Definition 3, Newton introduced the notion of *vis insita* as a latent force manifesting itself in different ways. In orbital motion *vis insita* manifests itself as a resistance to the force bending the body's orbit. The terms *reactio* and *vis contraria* are tied to this framework and are clearly linked to the third law of motion, a law repeatedly invoked by Newton in *Ex Epistola cujusdam ad Amicum*. Thus the third law is used by Newton in two quite separate conceptual frameworks. One is linked to the reciprocity of attractions, the other is that concerning centrifugal force that I have discussed in this paper.

This essay highlights important differences between Newton's conceptualization and the later accounts found in the classical mechanics with which we are familiar. I believe this process of conceptual defamiliarization to be an important ingredient of historical investigations.

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